Statistical Quality Control - Stat 3081

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Quality Control

- Industrial statistics deals with statistical methods most valuable in industry and with the important role they play in achieving high quality goods and services.

- Quality control is a powerful productivity technique for effective diagnosis of lack of conformity to settled standards in any of the materials, processes, machines or end products.

- Quality control, therefore, covers all the factors and processes of production which may be broadly classified as:

  - Quality of manpower
  - Quality of materials
  - Quality of machines
  - Quality of management

Quality Control - Contd.

- Controlling and improving quality has become an important business strategy for many organizations; manufacturers, distributors, transportation companies, financial services organizations; health care providers, and government agencies.
- Quality is a competitive advantage. A business that can delight customers by improving and controlling quality can dominate its competitors.
- This course is about the use of statistical methods and other problem-solving techniques to improve the quality of the products. These products consist of manufactured goods such as automobiles, computers, and clothing, as well as services such as the generation and distribution of electrical energy, public transportation, banking, health care, $\cdots$.

Dimensions of Quality

The quality of a product can be described and evaluated in eight different dimensions:

1. Performance (Will the product do the intended job?)
   - Potential customers usually evaluate a product to determine if it will perform certain specific functions and determine how well it performs them.
2. Reliability (How often does the product fail?)
   - Complex products, such as many appliances, automobiles, or airplanes, will usually require some repair over their service life. For example, you should expect that an automobile will require occasional repair, but if the car requires frequent repair, we say that it is unreliable.
3. Durability (How long does the product last?)
   - This is the effective service life of the product. Customers obviously want products that perform satisfactorily over a long period of time.

Dimensions of Quality - Contd.

4. Serviceability (How easy is it to repair the product?)
   - There are many industries in which the customers view of quality is directly influenced by how quickly and economically a repair or routine maintenance activity can be accomplished.
5. Aesthetics (What does the product look like?)
   - This is the visual appeal of the product, often taking into account factors such as style, color, shape and other sensory features.
6. Features (What does the product do?)
   - Usually, customers associate high quality with products that have added features; that is, those that have features beyond the basic performance of the competition. For example, you might consider a spreadsheet software package to be of superior quality if it had built-in statistical analysis features while its competitors did not.

Dimensions of Quality - Contd.

7. Perceived Quality (What is the reputation of the company or its product?)
    - In many cases, customers rely on the past reputation of the company concerning quality of its products. For example, if you make regular business trips using a particular airline, and the flight almost always arrives on time and the airline company does not lose or damage your luggage, you will probably prefer to fly on that carrier instead of its competitors.
8. Conformance to Standards (Is the product made exactly as the designer intended?)
    - Manufactured parts that do not exactly meet the designers requirements can cause significant quality problems. An automobile consists of several thousand parts. If each one is just slightly too big or too small, many of the components will not fit together properly, and the vehicle (or its major subsystems) may not perform as the designer intended.

Definition of Quality

1. The traditional definition of quality is based on the viewpoint that products and services must meet the requirements of customers.

Definition

Quality means fitness for use.

There are two general aspects of fitness for use:

1. **Quality of Design**: The intentional variations in grades or levels of quality. For example automobiles differ with respect to size, appearance, and performance. These differences are the result of intentional design differences among the types of automobiles.
2. **Quality of Conformance**: This is how well the product conforms to the specifications required by the design. Quality of conformance is influenced by a number of factors: the choice of manufacturing processes, the training and supervision of the workforce, the types of process controls, tests, and inspection activities, $\cdots$.

Definition of Quality - Contd.

2. The modern definition of quality is if variability in the important characteristics of a product decreases, the quality of the product increases.

Definition

Quality is inversely proportional to variability.

It also leads to the following definition of quality improvement:

Definition

Quality improvement is the reduction of variability in processes and products.

Excessive variability in process performance often results in waste (waste of money, time, and effort). Therefore, an alternate definition of quality improvement is the reduction of waste.

Quality Characteristics

- Every product possesses a number of elements that jointly describe what the user or consumer thinks of as quality.
- These parameters are often called quality characteristics (critical-to-quality (CTQ) characteristics).
- Quality characteristics may be of several types:
  - Physical: length, weight, voltage, viscosity/resistance
  - Sensory: taste, appearance, color
  - Time Orientation: reliability, durability, serviceability
- The different types of quality characteristics can relate directly or indirectly to the dimensions of quality.

Quality Characteristics - Contd.

- Since variability can only be described in statistical terms, statistical methods play a central role in quality improvement.
- Data on quality characteristics are typically classified as either attributes or variables.
    1. Variables data are usually continuous measurements, such as length, voltage, or viscosity/resistance.
    2. Attributes data are usually discrete data, often taking the form of counts.
- Quality characteristics are often evaluated relative to specifications.
- The specifications are the desired measurements for the quality characteristics of the components and subassemblies of the product, as well as the desired values for the quality characteristics in the final product.

Quality Characteristics - Contd.

- The desired value for quality characteristic is called the nominal or target value.
- The target value is usually bounded by a range of values that are sufficiently close to it.
    - The largest allowable value for a quality characteristic is called the upper specification limit (USL).
    - The smallest allowable value for a quality characteristic is called the lower specification limit (LSL).

Modeling Process Quality

- This section is just the review of probability theory (common discrete and continuous probability distributions).

FOR MORE YOU CAN LOOK AT THE TEXT BOOK.

Three Broad Categories of SQC

1. **Statistical Process Control (SPC)**: It is a collection of problem solving tools useful in achieving process stability and improving capability through the reduction of variability. A product should be produced by a process that is stable or repeatable. More precisely, the process must be operate with little variability around the target (nominal) value of the quality characteristics. A control chart is one of the primary techniques of SPC.

2. **Design of Experiment**: A designed experiment is the approach of determining the effect controllable input factors, on the product parameters, by systematically varying these factors in the process. It is extremely helpful in discovering the key variables influencing the quality characteristics of the product. Also, statistically designed experiments are invaluable in determining the levels of the controllable variables that optimize process performance.

Three Broad Categories of SQC - Contd.

3. **Acceptance Sampling**: It is the inspection and classification of a sample of units selected at random from a larger batch or lot and the ultimate decision about disposition of the lot. Inspection can occur at many points in a process, usually occurs at two points: incoming row materials or components, or final production.

Causes of Quality Variation

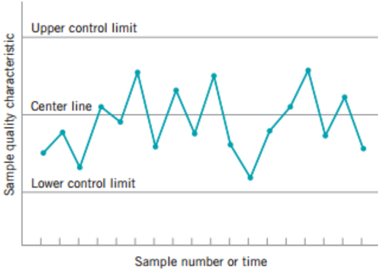
- **Random (Chance) Causes of Variation**:
  - In many production processes, a certain amount of inherent or natural variability will always exist.
  - For example, if you look at bottles of a soft drink in a grocery store, you will notice that no two bottles are filled to exactly the same level. Some are filled slightly higher and some slightly lower. These types of differences are completely normal. No two products are exactly alike because of slight differences in materials, workers, machines, tools, and other factors. This natural variability is called a "stable system of chance causes".
  - Such a variation is beyond the human control and cannot be prevented or eliminated under any circumstance.
  - Chance causes of variation is tolerable and does not affect the quality and utility of the product.
  - If there exist only chance causes of variation in a process, the process is said to be *in statistical control*.

Causes of Quality Variation - Contd.

- **Assignable Causes of Variation**:
  - The other kind of variation attributed to any production process is due to the non-random causes which affect the utility of the product.
  - Such variability may arise from three sources:
    - machines (improperly adjusted, needing repair, worn tool)
    - operators (poor employee training)
    - raw materials (defective)
  - Such sources of variability, which can be identified and eliminated, are called "assignable causes".
  - A process that is operating in the presence of assignable causes is said to be *out of control*.
  - The causes can be traced out from the type of defect observed in the product and the process is rectified.

Control Charts

- Statistical quality control (SQC) is a planned collection and effective use of data for studying causes of variations in quality.
- The objective of SPC is to detect the occurrence of assignable causes of process shifts in order to take immediate remedial (corrective) action before many nonconforming units are manufactured.
- A control chart is a widely used technique in process control. It is a graph that shows whether a sample of data falls within the common or normal range of variation.



Control Charts - Contd.

- The center line (CL) represents the average value of the quality characteristic corresponding to the *in-control state*.
- The horizontal lines above and below from the CL are the upper control limit (UCL) and lower control limit (LCL), respectively.
- If the points are within the control limits in a *random pattern*, then the process is assumed to be *in control* and no action is required.
- However, if there is a point outside of the control limits, this indicats that the process is *out of control*.
- Thus, an investigation to find the causes responsible for this behavior is required and a corrective action is needed to eliminate the assignable causes.

Control Charts for Variables

- A single measurable quality characteristic, such as a dimension, weight, or volume, is called a variable.
- When dealing with such a variable, it is necessary to monitor both the mean value of the quality characteristic and its variability.
- The *process average* or *mean quality level* is usually controlled with the control chart for means, or the $\bar{x}$ chart. The $\bar{x}$ chart is used to monitor changes in the mean of a process.
- The *process variability* can be monitored with either a control chart for the range, called an R chart, or a control chart for the standard deviation, called the S chart.
- Therefore, it is important to maintain control over both the process mean and process variability.

Control Charts for $\bar{x}$ and R

- Suppose that a quality characteristic (X) is normally distributed with mean μ and variance σ^2, where both μ and σ^2 are known.

- If $x_1, x_2, \cdots, x_n$ is a sample of size n, then the sample average is

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}.$$

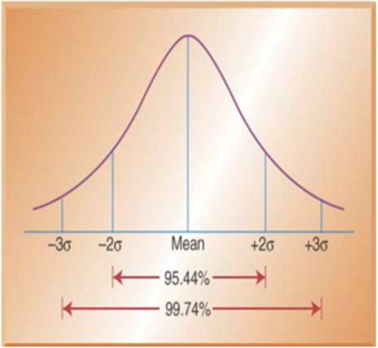
- Since $X \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

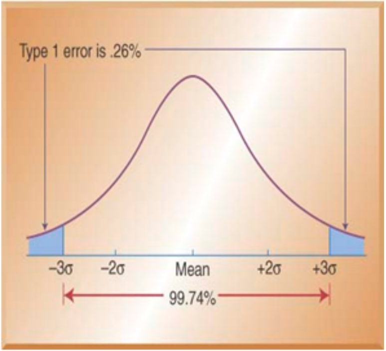
- The probability that any sample mean falls in the control limits is $(1 - \alpha)$. That is,

$$P\left(\mu - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \bar{X} \leq \mu + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha.$$

Control Charts for $\bar{x}$ and R - Contd.

- It is customary to replace $z_{\alpha/2}$ by 3, so that 3-sigma limits are employed (if $z_{\alpha/2} \Rightarrow \alpha = ?$).





Control Charts for $\bar{x}$ and R - Contd.

- Thus, the control limits for the $\bar{x}$ chart are:

$$\begin{aligned} LCL &= \mu - 3\frac{\sigma}{\sqrt{n}} = \mu - A\sigma \\ CL &= \mu \\ UCL &= \mu + 3\frac{\sigma}{\sqrt{n}} = \mu + A\sigma \end{aligned}$$

- And the control limits for the R chart are:

$$\begin{aligned} LCL &= d_2\sigma - 3d_3\sigma = (d_2 - 3d_3)\sigma = D_1\sigma \\ CL &= d_2\sigma \\ UCL &= d_2\sigma + 3d_3\sigma = (d_2 + 3d_3)\sigma = D_2\sigma \end{aligned}$$

- The terms A, D_1 and D_2 are constants that depend on n (are easily tabulated).

Control Charts for $\bar{x}$ and R - Contd.

- In practice, μ and σ^2 are not known. Therefore, they must be estimated from preliminary samples or subgroups taken when the process is thought to be in control.
- Suppose that m samples are taken, each containing n observations on the quality characteristic.
- Let $\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_m$ be the average of each sample. Then, the grand average:
$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m}$$
is the best estimator of μ, the process average.
- Let $R_1, R_2, \cdots, R_m$ be the ranges of the m samples. The average range is
$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m}.$$

Control Charts for $\bar{x}$ and R - Contd.

- m samples taken
- n observations each sample

Sample Number	Observations 1	2	$\cdots$	n	$\bar{x}$	R
1	x_{11}	x_{12}	$\cdots$	x_{1n}	$\bar{x}_1$	R_1
2	x_{21}	x_{22}	$\cdots$	x_{2n}	$\bar{x}_2$	R_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
m	x_{m1}	x_{m2}	$\cdots$	x_{mn}	$\bar{x}_m$	R_m
Average					$\bar{\bar{x}}$	$\bar{R}$

Control Charts for $\bar{x}$ and R - Contd.

- The control limits for the $\bar{x}$ chart are:

$$
\begin{aligned}
LCL &= \bar{\bar{x}} - 3\hat{\sigma}_{\bar{X}} = \bar{\bar{x}} - 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} - 3\frac{1}{\sqrt{n}d_2}\bar{R} = \bar{\bar{x}} - A_2\bar{R} \\
CL &= \bar{\bar{x}} \\
UCL &= \bar{\bar{x}} + 3\hat{\sigma}_{\bar{X}} = \bar{\bar{x}} + 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} + 3\frac{1}{\sqrt{n}d_2}\bar{R} = \bar{\bar{x}} + A_2\bar{R}
\end{aligned}
$$

- The control limits for the R chart are:

$$
\begin{aligned}
LCL &= \bar{R} - 3\hat{\sigma}_R = \bar{R} - 3\frac{d_3}{d_2}\bar{R} = \left(1 - 3\frac{d_3}{d_2}\right)\bar{R} = D_3\bar{R} \\
CL &= \bar{R} \\
UCL &= \bar{R} + 3\hat{\sigma}_R = \bar{R} + 3\frac{d_3}{d_2}\bar{R} = \left(1 + 3\frac{d_3}{d_2}\right)\bar{R} = D_4\bar{R}
\end{aligned}
$$

Ex 1: Inside diameter of piston rings - (page 252)

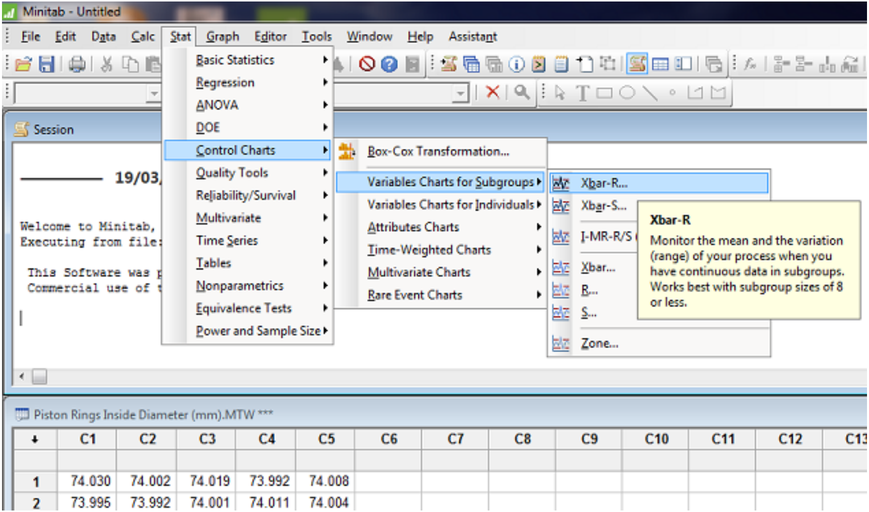
Sample Number	Observations				
1	74.030	74.002	74.019	73.992	74.008
2	73.995	73.992	74.001	74.011	74.004
3	73.988	74.024	74.021	74.005	74.002
4	74.002	73.996	73.993	74.015	74.009
5	73.992	74.007	74.015	73.989	74.014
6	74.009	73.994	73.997	73.985	73.993
7	73.995	74.006	73.994	74.000	74.005
8	73.985	74.003	73.993	74.015	73.988
9	74.008	73.995	74.009	74.005	74.004
10	73.998	74.000	73.990	74.007	73.995
11	73.994	73.998	73.994	73.995	73.990
12	74.004	74.000	74.007	74.000	73.996
13	73.983	74.002	73.998	73.997	74.012
14	74.006	73.967	73.994	74.000	73.984
15	74.012	74.014	73.998	73.999	74.007
16	74.000	73.984	74.005	73.998	73.996
17	73.994	74.012	73.986	74.005	74.007
18	74.006	74.010	74.018	74.003	74.000
19	73.984	74.002	74.003	74.005	73.997
20	74.000	74.010	74.013	74.020	74.003
21	73.982	74.001	74.015	74.005	73.996
22	74.004	73.999	73.990	74.006	74.009
23	74.010	73.989	73.990	74.009	74.014
24	74.015	74.008	73.993	74.000	74.010
25	73.982	73.984	73.995	74.017	74.013

Ex 1: Solution

- It is best to begin with the R chart. Because, unless process variability is in control, the process mean limits will not have much meaning.
- Given $m = 25$ and $n = 5$. We can easily obtain $\bar{\bar{x}} = 74.00118$ and $\bar{R} = 0.02324$.
- The control limits for the $\bar{x}$ chart:
  - $CL = \bar{\bar{x}} = 74.00118$
  - $LCL = \bar{\bar{x}} - A_2\bar{R} = 74.00118 - 0.577(0.02324) = 73.98777$
  - $UCL = \bar{\bar{x}} + A_2\bar{R} = 74.00118 + 0.577(0.02324) = 74.01458$
- The control limits for the $\bar{R}$ chart:
  - $CL = \bar{R} = 0.02324$
  - $LCL = D_3\bar{R} = 0(0.02324) = 0$
  - $UCL = D_4\bar{R} = 2.114(0.02324) = 0.04912$

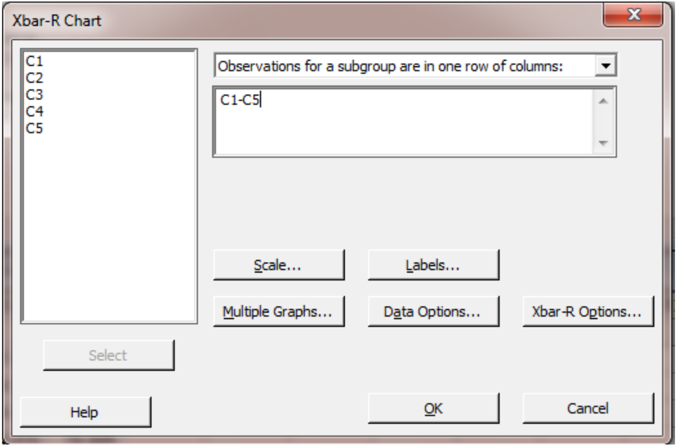
Ex 1: Solution - Contd.

Using Minitab

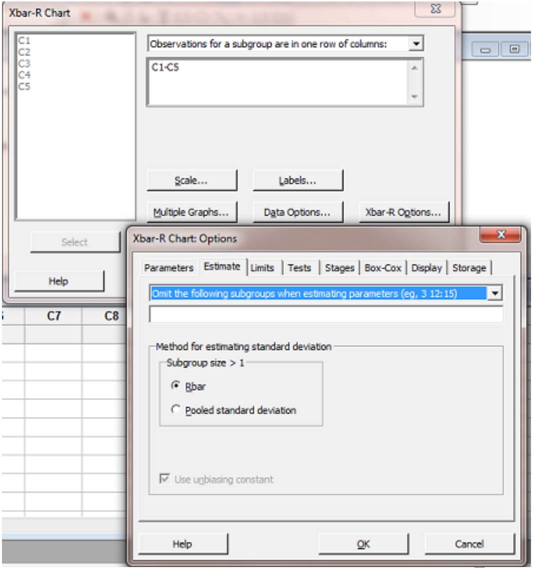


Ex 1: Solution - Contd.

- Then, the Xbar-R Chart window looks like:

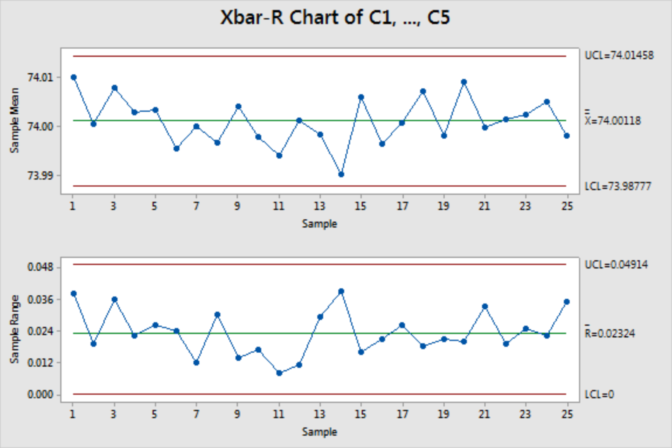


Ex 1: Solution - Contd.



Ex 1: Solution - Contd.

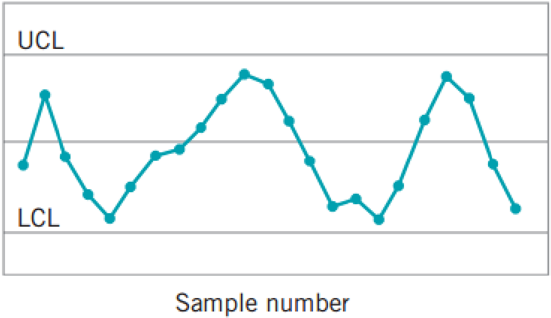
The control charts are:



Since both the $\bar{x}$ and R charts exhibit control, we would conclude that the process is in control at the stated levels and adopt the trial control limits for use in on-line statistical process control.

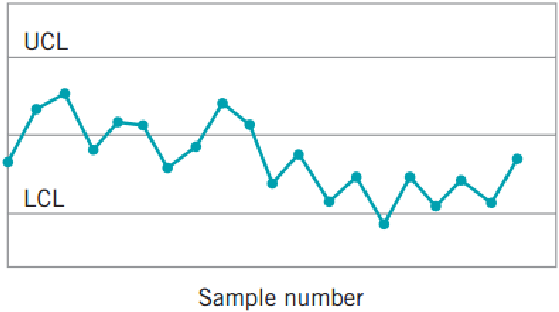
Control Charts - Contd.

- Even if all the points are inside the control limits, if they behave in a systematic or nonrandom manner, then this could be an indication that the process is *out of control*.
- **Cyclic Pattern**: may result from environmental changes (temperature), operator fatigue, regular rotation of operators and/or machines, or fluctuation in voltage or other variable.



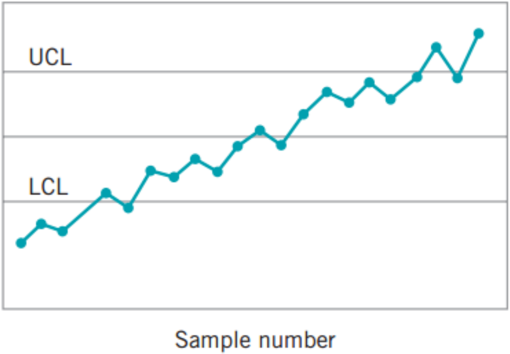
Control Charts - Contd.

- **Shift in process level**: The shift may result from the introduction of new workers, changes in methods (raw materials, machines), or a change in either the skill, attentiveness, or motivation of the operators.



Control Charts - Contd.

- **Trend**: A trend is a continuous movement in one direction. Trends are usually due to a gradual wearing out or deterioration of a tool or some other critical process component.

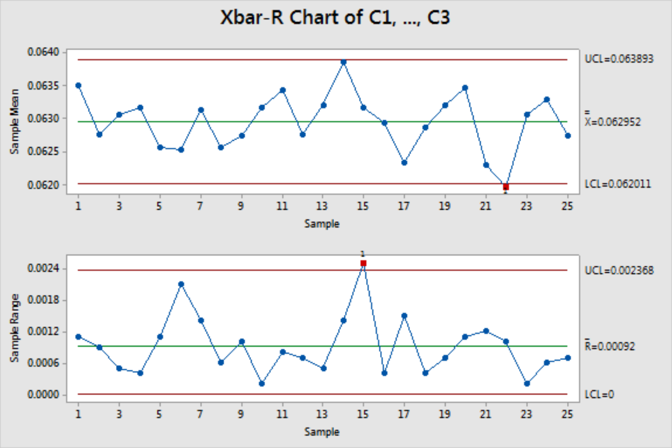


Ex 2: Board thickness - (page 273: TABLE 6E.4)

Sample Number	x_1	x_2	x_3
1	0.0629	0.0636	0.0640
2	0.0630	0.0631	0.0622
3	0.0628	0.0631	0.0633
4	0.0634	0.0630	0.0631
5	0.0619	0.0628	0.0630
6	0.0613	0.0629	0.0634
7	0.0630	0.0639	0.0625
8	0.0628	0.0627	0.0622
9	0.0623	0.0626	0.0633
10	0.0631	0.0631	0.0633
11	0.0635	0.0630	0.0638
12	0.0623	0.0630	0.0630
13	0.0635	0.0631	0.0630
14	0.0645	0.0640	0.0631
15	0.0619	0.0644	0.0632
16	0.0631	0.0627	0.0630
17	0.0616	0.0623	0.0631
18	0.0630	0.0630	0.0626
19	0.0636	0.0631	0.0629
20	0.0640	0.0635	0.0629
21	0.0628	0.0625	0.0616
22	0.0615	0.0625	0.0619
23	0.0630	0.0632	0.0630
24	0.0635	0.0629	0.0635
25	0.0623	0.0629	0.0630

Ex 2: Solution

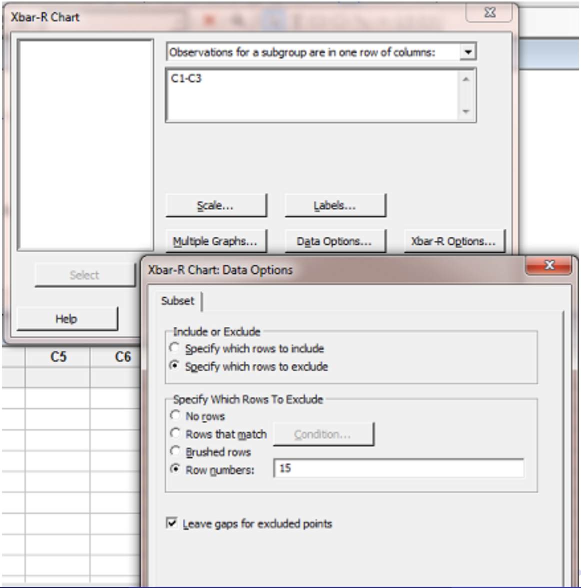
The control charts for the board thickness data are:



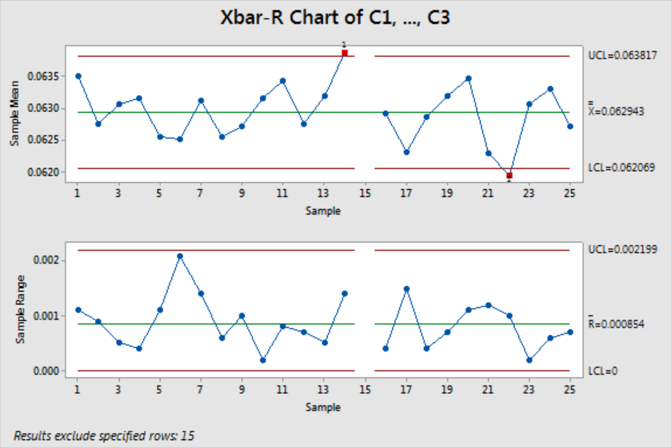
Ex 2: Solution - Contd.

- When the R chart is examined it is observed that the 15^{th} point is out of the control limits.
- That is, the process variability is out of control.
- We should examine this out of control point, looking for an assignable cause.
- If an assignable cause is found, the point is discarded and the trail control limits are recalculated, using only the remaining points.
- Suppose that an assignable cause is found for point 15. After discarding point 15, revised control limits are recalculated.

Ex 2: Solution - Contd.



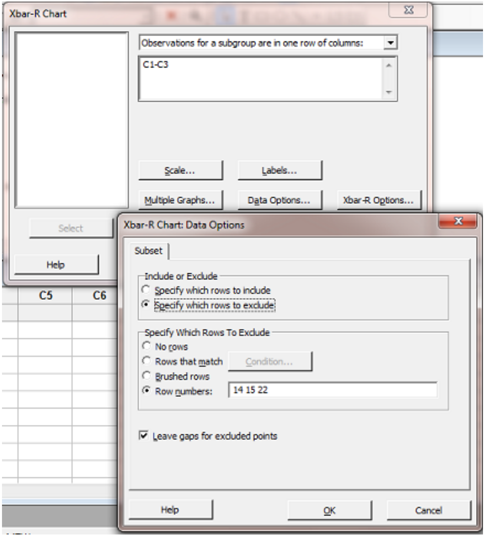
Ex 2: Solution - Contd.



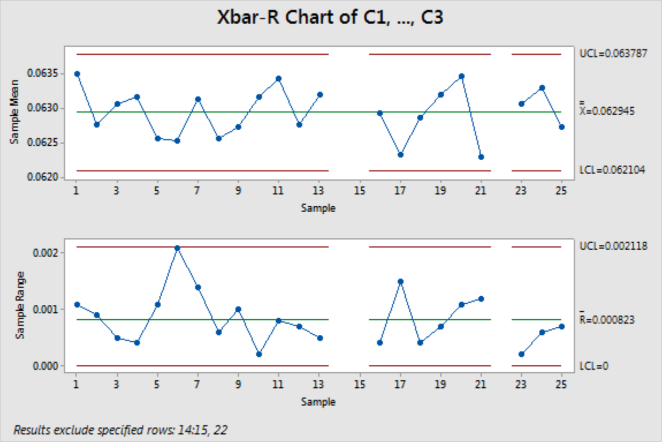
Ex 2: Solution - Contd.

- Now, process variability is in control when point 15 is discarded.
- However, points 14 and 22 are outside the limits in the $\bar{x}$ chart.
- That is, the process mean is out of control.
- Suppose that assignable cause is found for both of the out of control points.
- Then, we discard these points and recalculate the control limits for both $\bar{x}$ and R charts.

Ex 2: Solution - Contd.



Ex 2: Solution - Contd.



Both the revised charts indicate that process variability and mean are in control. Points 14, 15 and 22 are discarded assuming that an assignable cause is found for each point.

Control Charts for $\bar{x}$ and S

- Generally, $\bar{x}$ and S charts are preferable to their more familiar counterparts, $\bar{x}$ and R charts, when the sample size, n is moderately large, say $n > 10$.

- Recall that the range method for estimating σ loses statistical efficiency for moderate to large samples.

- $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator (UE) of σ^2.

- However, S is not an unbiased estimator of σ.

- If underlying distribution is normal, $E(S) = c_4\sigma$ (which is the CL for the S chart when the parameters are known) and $\sqrt{V(S)} = \sigma\sqrt{1 - c_4^2}$.

- The term c_4 is a constant depending on the sample size, n.

Control Charts for $\bar{x}$ and S - Contd.

- The control charts are:

$$LCL = c_4\sigma - 3\left(\sqrt{1-c_4^2}\right)\sigma = \left(c_4 - \sqrt{1-c_4^2}\right)\sigma = B_5\sigma$$
$$CL = c_4\sigma$$
$$UCL = c_4\sigma + 3\left(\sqrt{1-c_4^2}\right)\sigma = \left(c_4 + \sqrt{1-c_4^2}\right)\sigma = B_6\sigma$$

- If the parameters are unknown, the control limits for S chart are:

$$LCL = \bar{S} - 3\left(\sqrt{1-c_4^2}\right)\frac{\bar{S}}{c_4} = \left(1 - \left[\sqrt{1-c_4^2}\right]\frac{1}{c_4}\right)\bar{S} = B_3\bar{S}$$
$$CL = \bar{S}$$
$$UCL = \bar{S} + 3\left(\sqrt{1-c_4^2}\right)\frac{\bar{S}}{c_4} = \left(1 + \left[\sqrt{1-c_4^2}\right]\frac{1}{c_4}\right)\bar{S} = B_4\bar{S}$$

Control Charts for $\bar{x}$ and S - Contd.

- $E(S) = c_4\sigma \Rightarrow E(\bar{S}) = c_4\sigma$ where $\bar{S} = \dfrac{1}{m}\sum_{i=1}^{m} S_i$ where m is the number of preliminary samples, each of size n.
- $E\left(\dfrac{\bar{S}}{c_4}\right) = \sigma$, thus $\dfrac{\bar{S}}{c_4}$ is an unbiased estimator of σ.
- Control limits on the corresponding $\bar{x}$ chart

$$LCL = \bar{\bar{x}} - 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} - 3\frac{1}{c_4\sqrt{n}}\bar{S} = \bar{\bar{x}} - A_3\bar{S}$$

$$CL = \bar{\bar{x}}$$

$$UCL = \bar{\bar{x}} + 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} + 3\frac{1}{c_4\sqrt{n}}\bar{S} = \bar{\bar{x}} + A_3\bar{S}$$

Ex 3: Recall data given on Ex 1

- Can easily obtain $\bar{\bar{x}} = 74.00118$ and $\bar{S} = 0.00939$.
- The control limits for the $\bar{x}$ chart are:

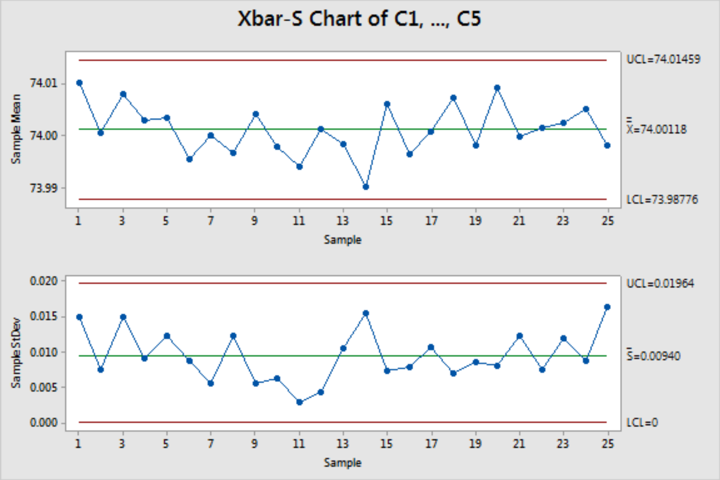
$$LCL = \bar{\bar{x}} - A_3\bar{S} = 74.00118 - 1.427(0.00939) = 73.98778$$
$$CL = \bar{\bar{x}} = 74.00118$$
$$UCL = \bar{\bar{x}} + A_3\bar{S} = 74.00118 + 1.427(0.00939) = 74.01458$$

- The control limits for the S chart are:

$$LCL = B_3\bar{S} = 0(0.00939) = 0$$
$$CL = \bar{S} = 0.00939$$
$$UCL = B_4\bar{S} = 2.089(0.00939) = 0.01962$$

Ex 3: Recall data given on Ex 1 - Contd.

- Using the $\bar{x}$ and S control charts:



Summary of the Control Charts

- Standards given:

Chart	Center Line	Control Limits
$\bar{x}$ (μ and σ known)	μ	$\mu \pm A\sigma$
R (σ known)	$d_2\sigma$	$LCL = D_1\sigma$ and $UCL = D_2\sigma$
S (σ known)	$c_4\sigma$	$LCL = B_5\sigma$ and $UCL = B_6\sigma$

- No standards given:

Chart	Center Line	Control Limits
$\bar{x}$ (using R)	$\bar{\bar{x}}$	$\bar{\bar{x}} \pm A_2 R$
$\bar{x}$ (using S)	$\bar{\bar{x}}$	$\bar{\bar{x}} \pm A_3 S$
R	$\bar{R}$	$LCL = D_3\bar{R}$ and $UCL = D_4\bar{R}$
S	$\bar{S}$	$LCL = B_3\bar{S}$ and $UCL = B_4\bar{S}$

Control Charts and Hypothesis Testing

- Suppose that the vertical axis is the sample average, $\bar{x}$. If the current value of $\bar{x}$ plots between the control limits, this indicates the process mean is in control; that is, it is equal to some value μ_0. If $\bar{x}$ exceeds either control limit, the process mean is out of control; that is, it is equal to some value $\mu_1 \neq \mu_0$.
    - $H_0 : \mu = \mu_0$ (process mean is in control)
    - $H_1 : \mu \neq \mu_0$ (process mean is out of control (a shift occurs))
- The control limits of the $\bar{x}$ chart are $\mu_0 \pm L\sigma_0$; L is the distance, of the control limits from the center line, in standard deviation units.
- The $CL = \mu_0$ (target or nominal value).
- If $LCL \leq \bar{x} \leq UCL$, conclude H_0 $(\mu = \mu_0)$.
- If $\bar{x} > UCL$ or $\bar{x} < LCL$, conclude H_1 $(\mu = \mu_1 \neq \mu_0)$.

Control Charts and Hypothesis Testing - Contd.

- **Type I Error**: Concluding the process is out of control when it is really in control.

$$\begin{aligned}\alpha &= P(\text{Type I Error}) \\ &= P(\bar{x} > UCL \text{ or } \bar{x} < LCL \ / \ H_0)\end{aligned}$$

- **Type II Error**: Concluding the process is in control when it is really out of control.

$$\begin{aligned}\beta &= P(\text{Type II Error}) \\ &= P(LCL \leq \bar{x} \leq UCL \ / \ H_1)\end{aligned}$$

OC Curve for an $\bar{x}$ Chart

- The ability of the $\bar{x}$ and R charts to detect shifts in process quality is described by their operating characteristic (OC) curves.
- OC Curve displays probability of type II error.
- This curve gives an indication of the ability of the control chart to detect process shifts of different magnitudes.
- Suppose a quality characteristic X is normally distributed with mean μ and variance σ^2. Then, $\bar{X} \sim N(\mu, \dfrac{\sigma^2}{n})$.
- If the mean shifts from the in-control value - say, μ_0 - to another value $\mu_1 = \mu_0 + k\sigma$, the probability of *not detecting this shift* (*not rejecting* H_0) on the first subsequent sample or the β-risk is:

$$\beta = P(LCL \leq \bar{X} \leq UCL \ / \ \mu = \mu_0 + k\sigma)$$

OC Curve for an $\bar{x}$ Chart - Contd.

- Thus,

$$\begin{aligned}\beta &= P(LCL \leq \bar{X} \leq UCL \ / \ \mu = \mu_0 + k\sigma) \\ &= P\left(\frac{LCL-\mu}{\sigma/\sqrt{n}} \leq Z \leq \frac{UCL-\mu}{\sigma/\sqrt{n}}\right) \\ &= P\left(\frac{LCL-[\mu_0+k\sigma]}{\sigma/\sqrt{n}} \leq Z \leq \frac{UCL-[\mu_0+k\sigma]}{\sigma/\sqrt{n}}\right)\end{aligned}$$

- Since $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$, the control limits are $\mu_0 \pm L\frac{\sigma}{\sqrt{n}}$,

$$\begin{aligned}\beta &= P\left(\frac{[\mu_0 - L\sigma/\sqrt{n}]-[\mu_0+k\sigma]}{\sigma/\sqrt{n}} \leq Z \leq \frac{[\mu_0 + L\sigma/\sqrt{n}]-[\mu_0+k\sigma]}{\sigma/\sqrt{n}}\right) \\ &= P(-L-k\sqrt{n} \leq Z \leq L-k\sqrt{n})\end{aligned}$$

Example 1

- Suppose we are using an $\bar{x}$ chart with the usual three-sigma limits. The sample size is $n = 5$. Determine the probability of not detecting a shift to $\mu_1 = \mu_0 + 2\sigma$ on the first sample following the shift.

- We have $L = 3$, $k = 2$ and $n = 5$. Thus,

$$\begin{aligned}
\beta &= P(-L - k\sqrt{n} \leq Z \leq L - k\sqrt{n}) \\
&= P(-3 - 2\sqrt{5} \leq Z \leq 3 - 2\sqrt{5}) = P(-7.47 \leq Z \leq -1.47) \\
&= P(0 \leq Z \leq 7.47) - P(0 \leq Z \leq 1.47) \\
&= 0.5 - 0.4292 = 0.0708.
\end{aligned}$$

- This is the β-risk, or the probability of not detecting such a shift.

- That is, the prob. of not detecting the shift on the first subsequent sample is 0.0708.

Example 1 - Contd.

- The prob. of that the shift will be detected on the first subsequent sample is $= 1 - \beta = 1 - 0.0708 = 0.9292$. (This is the prob. of one point being out of control if the process is actually out of control.)

- The prob. of such a shift will be detected on the second sample is

$$= \beta(1 - \beta) = 0.0658$$

- The prob. of such a shift will be detected on the third sample is

$$= \beta\beta(1 - \beta) = \beta^2(1 - \beta) = 0.0047$$

- The prob. of such a shift will be detected on the forth sample is

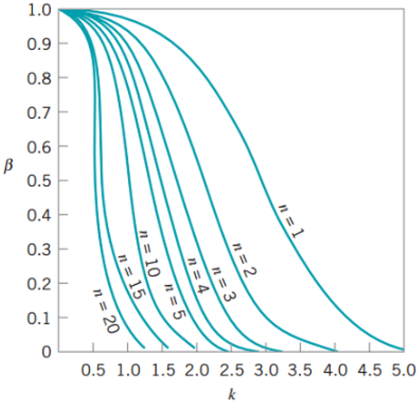
$$= \beta^3(1 - \beta) = 0.0003$$

- The prob. of such a shift will be detected on the r^{th} sample is

$$= \beta^{r-1}(1 - \beta) = (0.0708)^r(1 - 0.0708).$$

Sample OC Curve with the 3-sigma limits

- The OC curve (for the $\bar{x}$ chart) is constructed by plotting the β-risk (probability of not detecting the shift) against the magnitude of the shift (k) to be detected expressed in standard deviation units for different sample sizes n.



The Average Run Length (ARL)

- The function $\beta^{r-1}(1-\beta)$ is the probability distribution of a geometric random variable where the probability of success $p = 1 - \beta$.
- Recall that the geometric random variable (Y) is the number of trials required until a success is occurred $R_Y = \{1, 2, \cdots\}$. For our particular case, the success is *detecting the shift.*
- The expected number of samples taken until the shift is detected is simply called the *average run length* (ARL). It is the expected value of the geometric random variable: $E(Y) = \dfrac{1}{1-\beta}$.
- For the previous example, $ARL = \dfrac{1}{1-0.0708} = 1.076.$
- In other words, the ARL is the average number of points that must be plotted till a point indicates an out of control condition.

Example 2

- Consider an in-control process with mean $\mu = 300$ and standard deviation $\sigma = 3$. Subgroup of size 14 are used with control limits given by: $CL = \mu,\ LCL = \mu - A\sigma,\ UCL = \mu + A\sigma$. Suppose that a shift occurs in the mean and thus the new mean is $\mu_1 = 288$. Calculate the average number of samples required (following the shift) to detect an out of control situation.

- **Solution**: $n = 14$, $\mu = 300$ and $\sigma = 3.0$; $A = 3/\sqrt{14} = 0.8018$
  - $LCL = \mu - A\sigma = 300 - 0.8018(3) = 297.59$ and $UCL = \mu + A\sigma = 300 + 0.8018(3) = 302.41$.

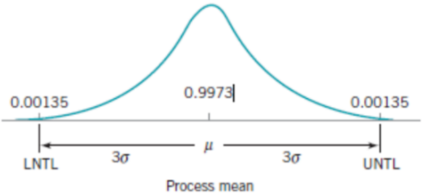
$$\begin{aligned}\beta &= P(LCL \leq \bar{X} \leq UCL \ / \ \mu = \mu_1 = 288) \\ &= P(297.59 \leq \bar{X} \leq 302.41 \ / \ \mu = \mu_1 = 288) \\ &= P\left(\frac{297.59 - 288}{3/\sqrt{14}} \leq Z \leq \frac{302.41 - 288}{3/\sqrt{14}}\right) \\ &= P(11.96 \leq Z \leq 17.97) = 0\end{aligned}$$

Example 2 - Contd.

- The probability of not detecting the shift in the first sample is $\beta = 0$.
- Thus, the probability of detecting the shift in the first subsequent (next) sample is $1 - \beta = 1 - 0 = 1$.
- The average number of samples required to detect an out of control situation is:
$$ARL = \frac{1}{1-\beta} = \frac{1}{1-0} = 1.$$
- The shift is detected almost with certainty in the next sample.

Control Limits and Specification Limits

- There is no relationship between the control limits and specification limits.
- The control limits are driven by the natural variability of the process (measured by the process standard deviation, σ), that is, by the natural tolerance limits of the process.
  - LNTL: 3σ below the process mean
  - UNTL: 3σ above the process mean



Control Limits and Specification Limits - Contd.

- The specification limits are determined externally. These may be set by the management, the manufacturing engineers, the customer, or by product developers/designers.
- **Product specifications**, often called **tolerances**, are preset ranges of acceptable quality characteristics.
- Sometimes, practitioners plot specification limits on the $\bar{x}$ control chart. This practice is completely incorrect and should not be done.
- When dealing with plots of individual observations (not averages), it is helpful to plot the specification limits on that chart.

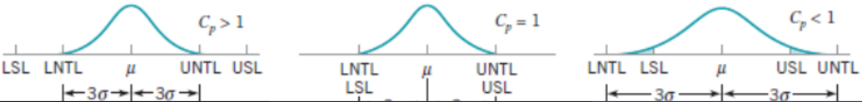
Process Capability

- In managing variables, the usual aim is not to achieve exactly the same quality level for each product, but to reduce the variation od products and and process parameters around a target value.
- In controlling a process, it is necessary to establish first that is in-statistical control, and then to compare its mean and variation with the specified target value and specification tolerance.
- To manufacture a product within a specification, the difference between USL and LSL must be less than the process variation.
- The ability of a production process to meet or exceed preset specifications is called process capability.
- A measure of the ability of the process to manufacture product that meets the specifications is called Process Capability Ratio (PCR).

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{2T}{6\sigma}.$$

Process Capability Ratio (PCR) - Contd.

- If $C_p < 1$, the process variation is greater than the specified tolerance so the process is not capable of producing within specifications. If $C_p = 1$, process variability just meets specifications (the process is minimally capable). For large values of C_p the process becomes increasingly capable as the process variability is tighter than specifications. That is, the process exceeds minimal capability.
- A C_p value of 1 means that 99.74% of the products produced fall within the specification limits which means that 0.26% (100-99.74)% of the products will not be acceptable (2600 wrong prescriptions out of a million).
- The C_p may be interpreted in terms of the proportion of tolerance interval used by process. The quantity $P = (1/C_p) \times 100\%$ is the percentage of the specification band that the process uses up.



Example 3

- Samples of $n = 5$ units are taken from a process every hour. The $\bar{x}$ and R values for a particular quality characteristics are determined as $\bar{\bar{x}} = 20$ and $\bar{R} = 4.56$ after 25 samples have been collected.
    - Find the 3-sigma control limits for the $\bar{x}$ and R charts.
    - Estimate the process standard deviation assuming that both charts exhibit control.
    - Assume that the process output is normally distributed. If specifications are 19 ± 5, then find the PCR and proportion of nonconforming products (defects). (The proportion of nonconforming products produced by the process to be determined here is called *Expected Within Performance* as the within standard deviation is to be used.)
    - If process mean shifts to 24, what is the probability of not detecting this shift on the first subsequent sample? Also find the *ARL*.

Example 3: Solution

- Control limits of $\bar{x}$ and R charts:
  - For $\bar{x}$ Chart: $CL = \bar{\bar{x}} = 20$
    - $LCL = \bar{\bar{x}} - A_2\bar{R} = 20 - 0.577(4.56) = 17.37$
    - $UCL = \bar{\bar{x}} + A_2\bar{R} = 20 + 0.577(4.56) = 22.63$
  - For R Chart: $CL = \bar{R} = 4.56$
    - $LCL = D_3\bar{R} = 0(4.56) = 0$
    - $UCL = D_4\bar{R} = 2.115(4.56) = 9.64$
- Estimated standard deviation: $\hat{\sigma} = \bar{R}/d_2 = 4.56/2.326 = 1.96$

Example 3 - Contd.

- We have given: $LSL = 19 - 5 = 14$ and $USL = 19 + 5 = 24$. Thus,

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{24 - 14}{6(1.96)} = \frac{2(5)}{6(1.96)} = 0.85$$

- The process uses $(1/0.85) \times 100\% = 117.6\%$ of the specification band. (The process is not capable.)

- **Expected Within Performance**: The proportions of nonconforming products is:

$$\begin{aligned} p &= P(X < LSL \text{ or } X > USL) = P(X < 14) + P(X > 24) \\ &= P\left(Z < \frac{14-20}{1.96}\right) + P\left(Z > \frac{24-20}{1.96}\right) \\ &= P(Z < -3.06) + P(Z > 2.04) = 0.5 - 0.4793 = 0.0207 \end{aligned}$$

That is, 2.07% (20700 parts per million (PPM)) of the products produced will be outside of the specifications.

Example 3 - Contd.

- For the probability of not detecting the shift:

$$\begin{aligned}\beta &= P(17.37 \leq \bar{X} \leq 22.63 \ / \ \mu = 24) \\ &= P\left(\frac{17.37 - 24}{1.96/\sqrt{5}} \leq Z \leq \frac{22.63 - 24}{1.96/\sqrt{5}}\right) \\ &= P(-7.56 \leq Z \leq -1.56) \\ &= P(0 \leq Z \leq 7.56) - P(0 \leq Z \leq 1.56) \\ &= 0.5 - 0.4406 \\ &= 0.0594\end{aligned}$$

The ARL is:

$$ARL = \frac{1}{1 - 0.0594} = 1.06$$

Example 4

- Recall Ex 1. Suppose that the specification limits for the inside diameter of piston rings is $74 \pm 0.05mm$. Find the PCR.

- Solution: $LSL = 73.95$ and $USL = 74.05$
  - $\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.023}{2.326} = 0.0099$
  - $\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{74.05 - 73.05}{6(0.0099)} = \frac{2(0.05)}{6(0.0099)} = 1.684$

- This implies that the natural tolerance limits (3σ above and below the mean) in the process are well inside the lower and upper specification limits.

- Or the process uses up to $(1/1.684) \times 100\% = 59.4\%$ of the specification band. (The process is capable is capable of producing products within specifications.)

One-Sided PCR

- PCR (C_p) assume that the process has both upper and lower specification limits.
- For one sided specifications, one sided process capability ratios are used.
  - Lower Specification only: $C_{pl} = \dfrac{\mu - LSL}{3\sigma}$
  - Upper specification only: $C_{pu} = \dfrac{USL - \mu}{3\sigma}$

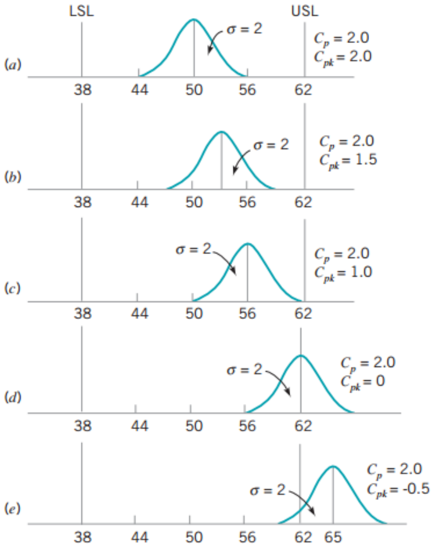
Measure of Actual Capability

- C_p does not take into account where the process mean is located (process centering) relative to the specifications.
- C_p simply measures the spread of the specifications relative to the six-sigma spread in the process.
- C_{pk} takes process centering into account. It is a one-sided PCR for specification limit nearest to process average.

$$C_{pk} = \min(C_{pl}, C_{pu})$$

- If $C_p = C_{pk}$, the process is centered at the midpoint of the specifications, and when $C_{pk} < C_p$ the process is off-center.
- The magnitude of C_{pk} relative to C_p is a direct measure of how off-center the process is operating.
- Thus, C_p measures potential capability in the process, whereas C_{pk} measures actual capability.

Relationship of C_p and C_{pk}



Example

- Consider the process shown in the above figure (b). Find the C_{pk}.
- Solution:
  - $C_{pl} = \frac{\mu - LSL}{3\sigma} = \frac{53 - 38}{3(2)} = 2.5$
  - $C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{62 - 53}{3(2)} = 1.5$
  - $C_{pk} = min(C_{pl}, C_{pu}) = min(2.5, 1.5) = 1.5$
  - Since $C_{pk} = 1.5 < C_p = 2.0$ the process is off-center.

Process Performance Indices (P_p and P_{pk})

- Process performance indices are defined as: $\hat{P}_p = \dfrac{USL - LSL}{6S}$.
- One sided indices are: $\hat{P}_{pl} = \dfrac{\hat{\mu} - LSL}{3S}$ and $\hat{P}_{pu} = \dfrac{USL - \hat{\mu}}{3S}$
- Taking centering into account: $\hat{P}_{pk} = \min(\hat{P}_{pl}, \hat{P}_{pu})$ where

$$S = \sqrt{\frac{1}{nm-1}\sum_{i=1}^{nm}(x_i - \bar{x})^2} \text{ and } \bar{x} = \frac{1}{nm}\sum_{i=1}^{nm} x_i.$$

- When the process is normally distributed and in control, $\hat{P}_p$ is essentially $\hat{C}_p$ and $\hat{P}_{pk}$ is essentially $\hat{C}_{pk}$.
- Now the fraction of defective units when the overall standard deviation is used is called the *Expected Overall Performance*.
- But, if the process is not in control, P_p and P_{pk} have *no meaning*.

Ex 3: Glass Container Bursting-Strength - (Page 365)

Sample	Data					$\bar{x}$	R
1	265	205	263	307	220	252.0	102
2	268	260	234	299	215	255.2	84
3	197	286	274	243	231	246.2	89
4	267	281	265	214	318	269.0	104
5	346	317	242	258	276	287.8	104
6	300	208	187	264	271	246.0	113
7	280	242	260	321	228	266.2	93
8	250	299	258	267	293	273.4	49
9	265	254	281	294	223	263.4	71
10	260	308	235	283	277	272.6	73
11	200	235	246	328	296	261.0	128
12	276	264	269	235	290	266.8	55
13	221	176	248	263	231	227.8	87
14	334	280	265	272	283	286.8	69
15	265	262	271	245	301	268.8	56
16	280	274	253	287	258	270.4	34
17	261	248	260	274	337	276.0	89
18	250	278	254	274	275	266.2	28
19	278	250	265	270	298	272.2	48
20	257	210	280	269	251	253.4	70
						$\bar{\bar{x}} = 264.06$	$\bar{R} = 77.3$

Ex 3: - Contd.

- Estimate the process parameters from the control chart.
- Find the overall mean and standard deviation.
- If the lower specification limit on bursting strength is 200 *psi*,
  - find the overall (actual) and within (potential) capability.
  - find the observed performance.
  - find the expected overall and within performance.

Ex 3: Solution

- **Estimated Process Parameters**:
  - Estimated μ: $\bar{\bar{x}} = 264.06$
  - Estimated σ: $\hat{\sigma} = \bar{R}/d_2 = 77.3/2.326 = 33.23$
- **Overall Mean**: $\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_{100}}{100} = 264.06$
- **Overall S.D**: $S = \sqrt{\dfrac{1}{100-1}\sum_{i=1}^{100}(x_i - 264.06)^2} = 32.0179$
- $LSL = 200$
  - **Overall Capability**: $\hat{P}_{pl} = \dfrac{\hat{\mu} - LSL}{3S} = \dfrac{264.06 - 200}{3(32.02)} = 0.67$
  - **Within Capability**: $\hat{C}_{pl} = \dfrac{\hat{\mu} - LSL}{3\hat{\sigma}} = \dfrac{264.06 - 200}{3(33.23)} = 0.64$

Ex 3: Solution - Contd.

- **Observed Performance**: Of the total 100 glass containers, there are 3 containers with strength less than 200 *psi*. Thus, the observed fraction of defective containers is 3/100=0.03 $\Rightarrow$ 30000 PPM.

- **Expected Overall Performance**: The expected overall performance is the fraction of defective glass containers produced when the overall standard deviation is used.

$$\begin{aligned}
\hat{p} &= P(X < LSL) = P(X < 200) \\
&= P\left(Z < \frac{LSL - \mu}{\sigma}\right) = P\left(Z < \frac{200 - 264.06}{32.0179}\right) \\
&= P(Z < -2.000756) \\
&= 0.02270935 \Rightarrow 22709.35 \text{ PPM}
\end{aligned}$$

The estimated fallout is about 22709 nonconforming glass containers per million when the overall standard deviation is used.

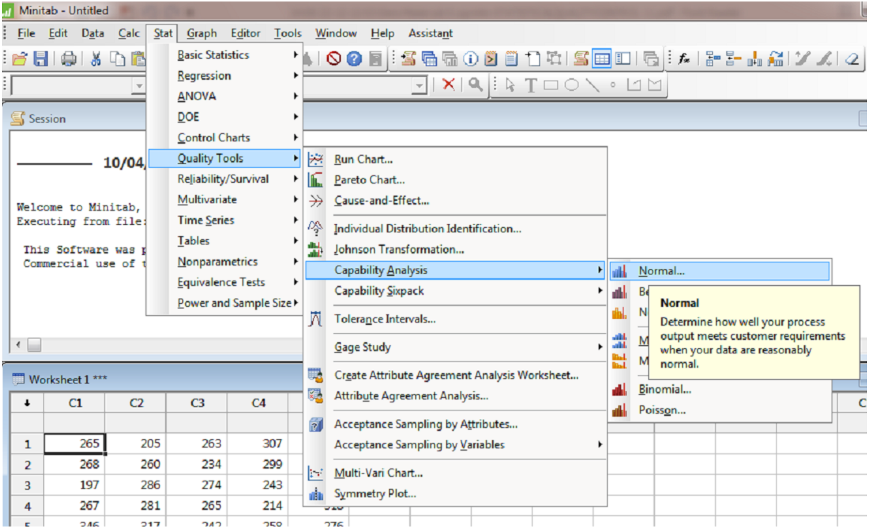
Ex 3: Solution - Contd.

- **Expected Within Performance**: The expected within performance is the fraction of defective glass containers produced when the within standard deviation is used.

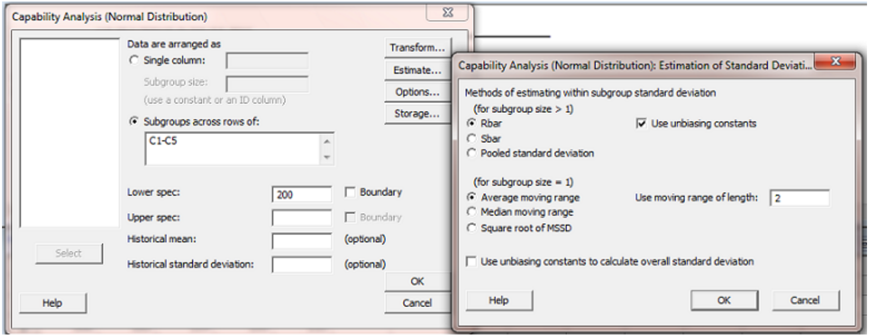
$$\begin{aligned}\hat{p} &= P(X < LSL) = P(X < 200) \\ &= P\left(Z < \frac{LSL - \mu}{\sigma}\right) = P\left(Z < \frac{200 - 264.06}{32.233}\right) \\ &= P(Z < -1.927602) \\ &= 0.02695232 \Rightarrow 26952.32 \text{ PPM}\end{aligned}$$

The estimated fallout is about about 26952 nonconforming glass containers per million when the within standard deviation is used.

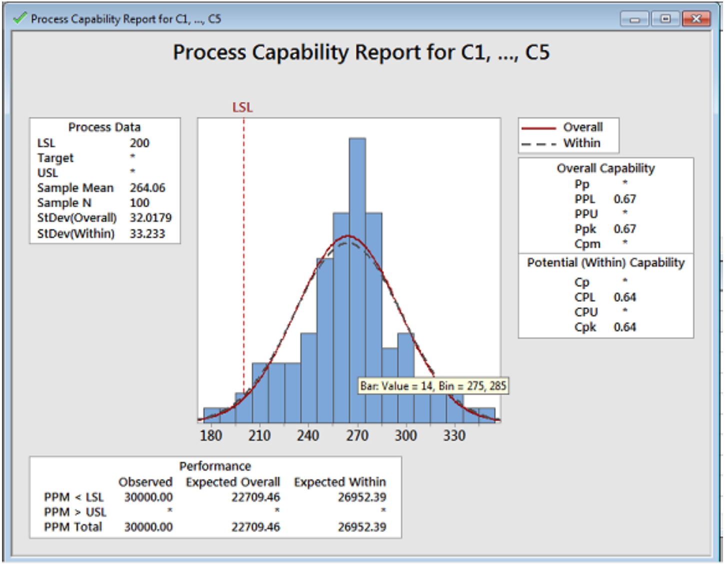
Ex 3: Minitab Solution



Ex 3: Minitab Solution - Contd.



Ex 3: Minitab Solution - Contd.



Attributes Data

- Many quality characteristics cannot be conveniently represented numerically.
- In such cases, usually each item inspected and classified as either conforming or nonconforming to the specifications on that quality characteristic.
- The terminology *defective* or *nondefective* is often used to identify these two classifications of product.
- Recently, the terminology *conforming* and *nonconforming* has become popular.

The p Chart: Fraction Nonconforming

- The p chart is used to measure the proportion of defectives in a sample.
- The *fraction nonconforming* is defined as the ratio of the number of nonconforming items in a population to the total number of items in that population.
- The items may have several quality characteristics that are examined simultaneously by the inspector.
- If the item does not conform to standard on one or more of these characteristics, it is classified as *nonconforming*.
- The statistical principles underlying the control chart for fraction nonconforming are based on the binomial distribution.

The p Chart: Fraction Nonconforming - Contd.

- Suppose the production process is operating in a stable manner and that successive units produced are independent.
- Let D denote the number of units of product that are nonconforming in a random sample of n units.
- Let p be the probability that any unit will be a defective or nonconforming (not conform to specifications).
- $D \sim Bin(n, p) \Rightarrow P(D = x) = \binom{n}{x} p^x (1-p)^{n-x}, \; x = 0, 1, 2, \cdots, n$
- $E(D) = np$ and $V(D) = np(1-p)$
- Estimator of p: Sample fraction nonconforming $\hat{p} = \dfrac{D}{n}$.
- $E(\hat{p}) = p$ and $V(\hat{p}) = \dfrac{p(1-p)}{n}$.

The p Chart: Fraction Nonconforming - Contd.

- Suppose that the true fraction nonconforming p in the production process is known or is a specified standard value.
- Then, the center line and control limits of the fraction nonconforming control chart are:
  - $CL = p$
  - $LCL = p - 3\sqrt{\frac{p(1-p)}{n}}$
  - $UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$
- If the lower control limit $LCL < 0$, set $LCL = 0$.

The p Chart: Fraction Nonconforming - Contd.

- When the process fraction nonconforming p is not known, then it must be estimated from m preliminary samples, each of size n.
- The fraction nonconforming in the i^{th} sample is the ratio of the number of nonconforming units in the i^{th} sample (D_i) to the size of the sample (n). That is, $\bar{p} = \frac{1}{mn}\sum_{i=1}^{m} D_i = \frac{1}{m}\sum_{i=1}^{m} \hat{p}_i$.
- The control limits of the p control chart are:
  - $CL = \bar{p}$
  - $LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
  - $UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Ex 4: Orange Juice Cans Leak - (Page 293)

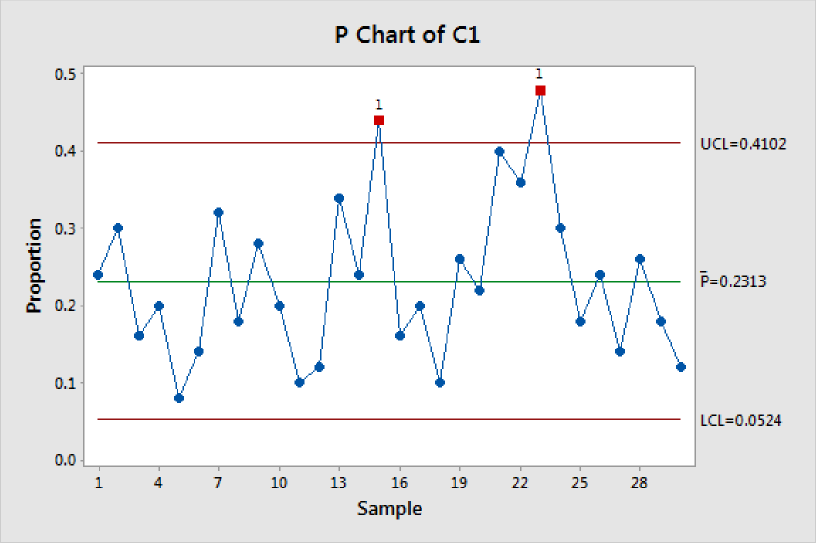
- 30 samples each with sample size 50. Construct the p chart.

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

Ex 4: Solution

- $\hat{p} = \dfrac{1}{mn}\sum_{i=1}^{m} D_i = \dfrac{347}{30(50)} = 0.2313$
- The control limits are:
  - $CL = \bar{p} = 0.2313$
  - $LCL = \bar{p} - 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 0.2313 - 3\sqrt{\dfrac{0.2313(1-0.2313)}{50}} = 0.0524$
  - $UCL = \bar{p} + 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 0.2313 + 3\sqrt{\dfrac{0.2313(1-0.2313)}{50}} = 0.4102$

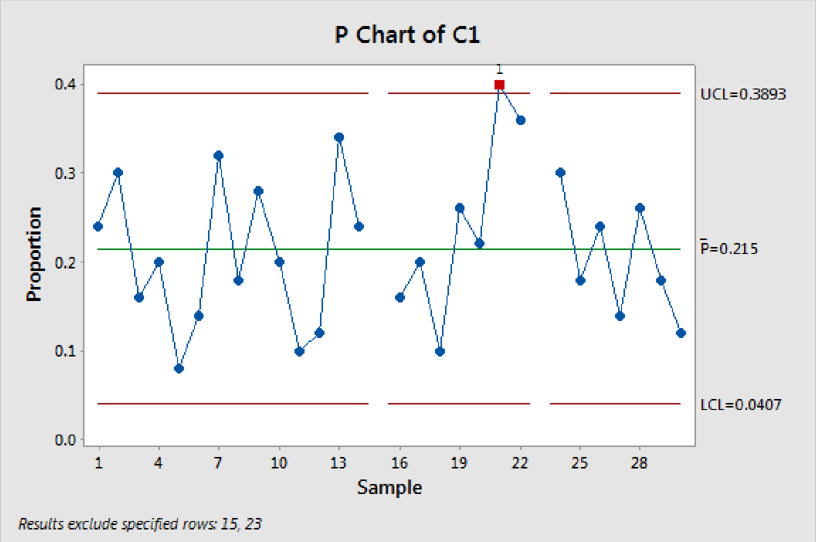
Ex 4: Solution - Contd.



Ex 4: Solution - Contd.

- Note that two points, those from samples 15 and 23, plot above the upper control limit, so the process is not in control.
- These points must be investigated to see whether an assignable cause can be determined.
- Consequently, samples 15 and 23 are eliminated, and the new center line and revised control limits are calculated as:
  - $CL = \bar{p} = \frac{1}{mn}\sum_{i=1}^{m} D_i = \frac{301}{28(50)} = 0.215$
  - $LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.215 - 3\sqrt{\frac{0.215(1-0.215)}{50}} = 0.0407$
  - $UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.215 + 3\sqrt{\frac{0.215(1-0.215)}{50}} = 0.3893$

Ex 4: Solution - Contd.



Ex 4: Solution - Contd.

- Note also that the fraction nonconforming from sample 21 now exceeds the upper control limit.
- However, analysis of the data does not produce any reasonable or logical assignable cause for this, and the point can be retained.
- Therefore, the new control limits can be used for future samples.

The np Chart: Number Nonconforming

- It is also possible to base a control chart on the number nonconforming rather than the fraction nonconforming.
- Number of nonconforming items among n items: $D \sim Bin(n, p)$, $E(D) = np$ and $V(D) = np(1-p)$.
- The control limits are:
  - $CL = np$
  - $LCL = np - 3\sqrt{np(1-p)}$
  - $UCL = np + 3\sqrt{np(1-p)}$
- If the standard value of p is not available, then $\bar{p}$ can be used to estimate p.

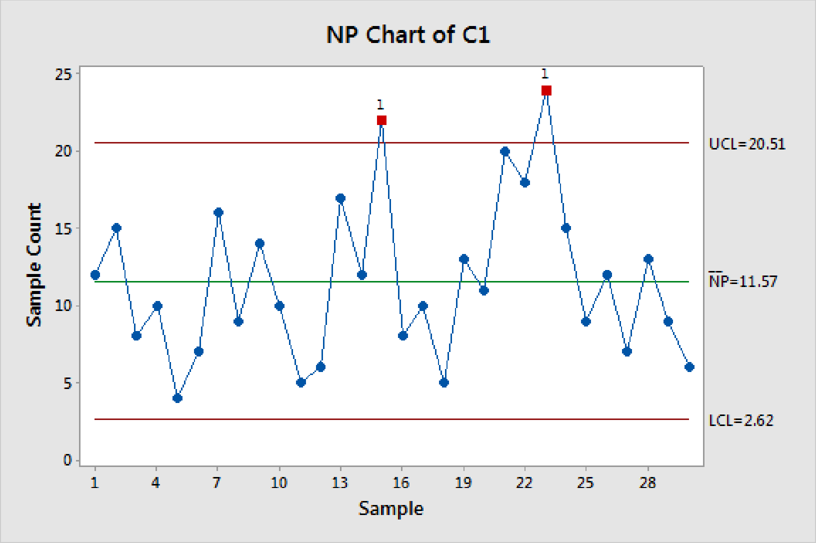
Example: Recall Ex 4

- Obtain the control limits for an np chart.

Solution:

- $CL = n\bar{p} = 50(0.2313) = 11.565$
- $LCL = n\bar{p} - 3\sqrt{np(1-p)} = 11.565 - 3\sqrt{11.565(1-0.2313)} = 2.62$
- $UCL = \bar{p} + 3\sqrt{np(1-p)} = 11.565 + 3\sqrt{11.565(1-0.2313)} = 20.51$

Example: Recall Ex 4 - Contd.



The OC Curve

- The operating-characteristic (or OC) function of the fraction nonconforming control chart is a graphical display of the probability of incorrectly accepting the hypothesis of statistical control (i.e., a type II error) against the process fraction nonconforming.

- The β risk is:

$$\begin{aligned}\beta &= P(LCL < \hat{p} < UCL/p) \\ &= P(\hat{p} < UCL/p) - P(\hat{p} \leq LCL/p) \\ &= P(D < nUCL/p) - P(D \leq nLCL/p) \\ &= \sum_{x=0}^{<nUCL} \binom{n}{x} (p)^x (1-p)^{n-x} - \sum_{x=0}^{\leq nLCL} \binom{n}{x} (p)^x (1-p)^{n-x}\end{aligned}$$

- Since D is a binomial random variable, β is obtained from the cumulative binomial distribution.

Example

- Given $n = 50$ and the control limits for the p chart as $LCL = 0.0303$ and UCL=0.3691. Find β if $p = 0.15$.

- **Solution**:

$$
\begin{aligned}
\beta &= P(nLCL < D < nUCL/p = 0.15) \\
&= P(D < 50(0.3697)/p = 0.15) - P(D \leq 50(0.0303)/p = 0.15) \\
&= P(D < 18.49/p = 0.15) - P(D \leq 1.52/p = 0.15) \\
&= P(D \leq 18/p = 0.15) - P(D \leq 1/p = 0.15) \\
&= \sum_{x=0}^{18} \binom{50}{x} (0.15)^x (0.85)^{50-x} - \sum_{x=0}^{1} \binom{50}{x} (0.15)^x (0.85)^{50-x} \\
&= 0.99994 - 0.0029 \\
&= 0.9970
\end{aligned}
$$

The c Chart: Number of Nonconformities in a Unit

- It is to develop control charts for the total number of nonconformities in a unit. The c chart is used to monitor the number of defects per unit.
- This control chart assume that the occurrence of nonconformities in a unit has a Poisson distribution.
- $P(x) = \frac{e^{-c}c^x}{x!}$ where x is the number of nonconformities in a unit.
- $E(X) = c = V(X)$
- The control limits for the c chart are:
  - $CL = c$
  - $LCL = c - 3\sqrt{c}$
  - $UCL = c + 3\sqrt{c}$
- If $LCL < 0$, set it zero.

The c Chart: - Contd.

- If c is not known, the control limits for the c chart are:
  - $CL = \bar{c}$
  - $LCL = \bar{c} - 3\sqrt{\bar{c}}$
  - $UCL = \bar{c} + 3\sqrt{\bar{c}}$
- Here $\bar{c} = \frac{1}{m}\sum_{i=1}^{m} c_i$ where c_i is the number of nonconformities in the i^{th} inspection unit.

Ex 5: Circuit Board Nonconformities - (Page 310)

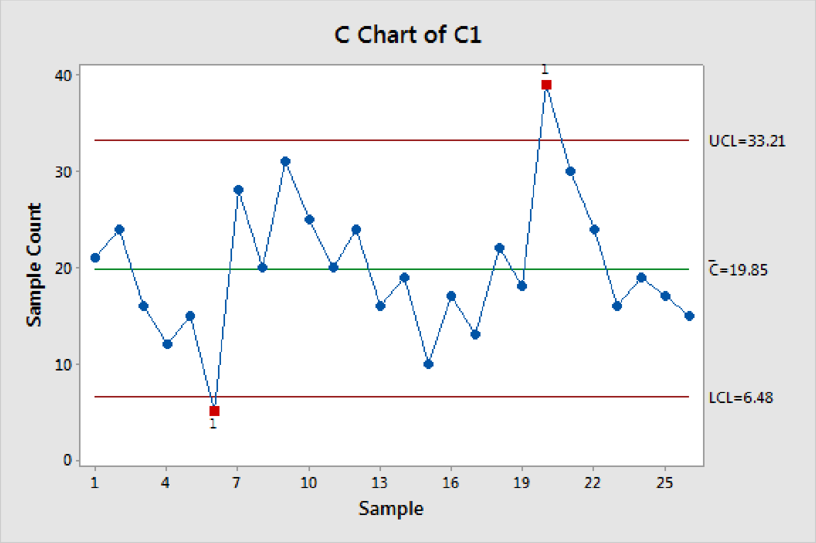
- Construct the c chart for the nonconformities in one inspection unit (100 printed circuit boards).

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

Ex 5: Solution

- $\bar{c} = \frac{1}{m} \sum_{i=1}^{m} c_i = \frac{516}{26} = 19.85$
- The control limits are:
  - $CL = \bar{c} = 19.85$
  - $LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$
  - $UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$

Ex 5: Solution - Contd.



The u Chart: Average Number of Nonconformities per Unit

- If there are x nonconformities in a sample of n inspection units, then the average number of nonconformities per unit is $u = \frac{x}{n}$.
- $X \sim Poisson(\lambda) \Rightarrow E(X) = \lambda = V(X)$
- $E(U) = \frac{\lambda}{n}$ is est. by $\bar{u} = \frac{1}{m}\sum_{i=1}^{m} u_i$ and $V(U) = \frac{\lambda}{n^2}$ is est. by $\frac{\bar{u}}{n}$
- The control limits for u chart are:
  - $CL = \bar{u}$
  - $LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$
  - $UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$

Example concerning the u chart

LOOK AT THE TEXT BOOK.

Lot Disposition

- One aspect of quality assurance is the inspection of raw materials or finished products. When inspection is for the purpose of accepting or rejecting a batch of products based on adherence to standards, it is called *acceptance sampling* (*sampling inspection*).
- Acceptance sampling is the second important field of quality control.
- The procedure is to select a random sample of products from the lot or batch and then some quality characteristics of the units in the sample are inspected for a decision to either accept or reject the entire lot. This is decision called *lot disposition* (*lot sentencing*).
- Accepted lots are put into production; rejected lots may be returned to the supplier or may be subjected to some other lot disposition action.

Three Approaches to Lot Disposition/Sentencing

- **Accept with no inspection**: Useful in situations where either the supplier's process is so good or where there is no economic justification to look for defective units. For example, if the supplier's process capability ratio is 3 or 4, inspection is unlikely to discover any defective units.
- **Complete (100%) Inspection**: This is inspecting every item in the lot and then removing all defective units found (replaced with good ones). Useful in situations where the component is extremely critical and passing any defectives would result in a high failure cost at subsequent stages, or where the supplier's process capability is inadequate to meet specifications.
- **Acceptance Sampling**: It is the process of randomly inspecting a certain number of items from a lot or batch in order to decide whether to accept or reject the entire batch.

Acceptance Sampling

- What makes acceptance sampling different from statistical process control is that acceptance sampling is performed either before or after the process, rather than during the process.
- That is, acceptance sampling is meant for product control while control charts are meant for process control.
- Acceptance sampling is useful when:
  - the cost of 100% inspection is very expensive.
  - inspecting every item is not physically possible.
  - testing of items is of destructive nature (testing eggs for salmonella).
  - the producer has an excellent quality history, no need to go for 100% inspection.

Sampling Plans

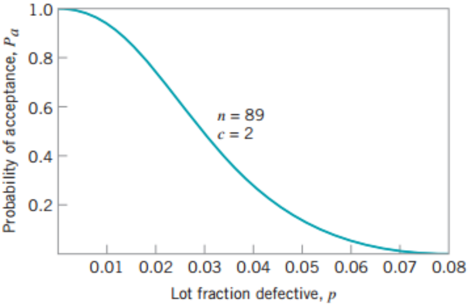
- A sampling plan is a plan for acceptance sampling that precisely specifies the parameters of the sampling process and the acceptance/rejection criteria.
- Each item in the sample is examined and is labeled as either "good" or "bad". Depending on the number of defects or "bad" items found, the entire lot is either accepted or rejected.
- Based on the number of samples required for a decision, there are different types of sampling plans.
  - Single sampling plans
  - Double sampling plans
  - Multiple sampling plans
  - Sequential sampling plans

Single Sampling Plan

- A single sampling plan is a lot sentencing procedure in which a single sample of n units is selected from the lot and the disposition of the lot is determined based on the basis of the sample information. It is determined by three parameters: size of the lot N, size of the sample n and acceptance number c.
- The plan of action is to select a sample of n units from the lot and count the number of defectives d, and then if $d \leq c$ accept the lot and if $d > c$ reject the lot.
- Different sampling plans have different capabilities for discriminating between good and bad lots. At one extreme is 100 percent inspection, which has perfect discriminating power.
- However, as the size of the sample inspected decreases, so does the chance of accepting a defective lot.

The OC Curve

- The performance (discriminatory power) of an acceptance-sampling plan can be shown by the operating characteristic (OC) curve.
- The OC curve plots the probability of accepting the lot given various proportions of defects in the lot. The OC curve of the sampling plan $n = 89$, $c = 2$ is shown below.



How the points on this curve are obtained?

- Suppose that the lot size N is large (theoretically infinite). Let D denote the number of defectives in a random sample of n items.
- Thus $D \sim Bin(n, p)$ where p is the fraction of defective items in the lot.

$$P(D = d) = \binom{n}{d} p^d (1-p)^{n-d};\ d = 0, 1, 2, \cdots, n$$

- Therefore, the probability of accepting the lot is:

$$P_a = P(d \leq c) = \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d}$$

Example

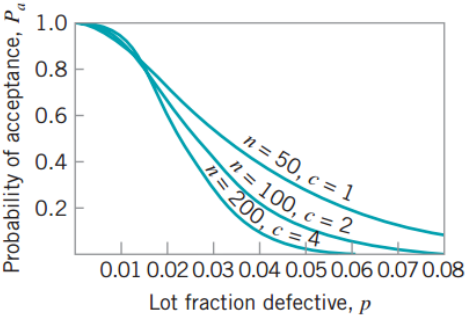
- Calculate the acceptance probability for the single sampling plan $n = 89$ and $c = 2$ if $p = 0.01$.

$$
\begin{aligned}
P(d \leq 2) &= \sum_{d=0}^{2} \binom{89}{d} (0.01)^d (1 - 0.01)^{89-d} \\
&= \binom{89}{0} (0.01)^0 (1 - 0.01)^{89-0} + \binom{89}{1} (0.01)^1 (1 - 0.01)^{89-1} \\
&+ \binom{89}{2} (0.01)^2 (1 - 0.01)^{89-2} = 0.9397
\end{aligned}
$$

- If the lots are 1% defective ($p = 0.01$), the probability of acceptance is approximately 0.94.

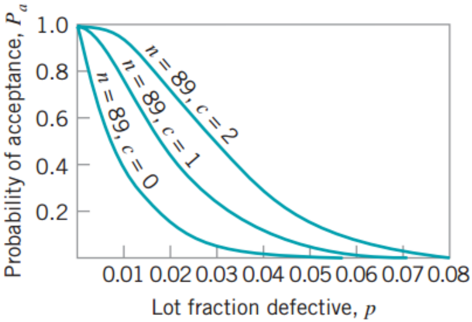
- This means that if 100 lots from a process that manufactures 1% defective product are submitted to this sampling plan, we will expect to accept 94 of the lots and reject 6 of them.

Effect of n on OC Curves



- The OC curve becomes more idealized as the sample size increases.
- Note that the acceptance number c is kept proportional to n.
- The greater the slope of the OC curve, the greater the discriminatory power.

Effect of c on OC Curves



- Changing the acceptance number c does not dramatically change the slope of the OC curve.
- As the acceptance number is decreased, the OC curve is shifted to the left.

OC Curve for Small Lot Size

- When the lot size (N) is small, hypergeometric distribution is used for calculating the probability of lot acceptance and accordingly plotting the OC curve.

$$P_a = P(D \leq c) = \sum_{d=0}^{c} \frac{\binom{Np}{d}\binom{N(1-p)}{n-d}}{\binom{N}{n}}$$

- Example: $N = 500$, $n = 50$, $c = 1$, For given $p = 0.01$,

$$\begin{aligned} P(D \leq c) &= \frac{\binom{500(0.01)}{0}\binom{500(0.99)}{50}}{\binom{500}{50}} + \frac{\binom{500(0.01)}{1}\binom{500(0.99)}{49}}{\binom{500}{50}} \\ &= 0.5890 + 0.3303 \\ &= 0.9193 \end{aligned}$$

OC Curve for Small Lot Size - Contd.

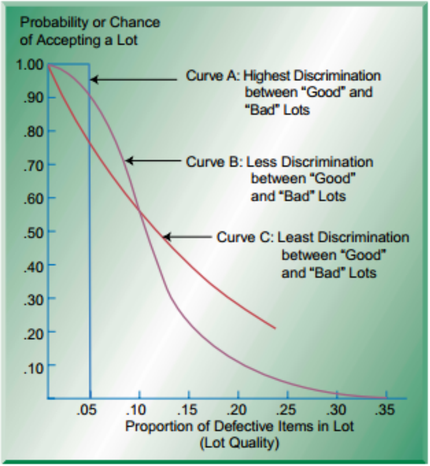
- Suppose that $N = 500$ is considered large. The probability of lot acceptance is found by using the binomial distribution.

$$\begin{aligned} P(d \leq 1) &= \sum_{d=0}^{1} \binom{50}{d} (0.01)^d (1-0.01)^{50-d} \\ &= \binom{50}{0} (0.01)^0 (0.99)^{50} + \binom{50}{1} (0.01)^1 (0.99)^{49} \\ &= 0.9106 \end{aligned}$$

- Whenever $n/N \leq 0.10$, the OC curves using both distributions are almost same.

Acceptable Quality Level (AQL)

- Regardless of the type of sampling plan selected, there is still a chance of accepting "bad" lots and rejecting "good" lots.



Acceptable Quality Level (AQL) - Contd.

- The steeper the OC curve, the better the sampling plan is for discriminating between "good" and "bad".
- When 100% inspection is not possible, there is a certain amount of risk (for consumers) in accepting defective lots and a certain amount of risk (for producers) in rejecting good lots.
- There is a small percentage of defects that consumers are willing to accept the lot which is called the **acceptable quality level (AQL)**.
- However, sometimes the percentage of defects that passes through is higher than the AQL.

Acceptable Quality Level (AQL) - Contd.

- Consumers usually tolerate a few more defects in a single lot until the number of defects reaches a threshold level beyond which they will not tolerate them.
- This threshold level is called the **lot tolerance percent defective (LTPD)/ rejectable quality level (RQL)/limiting quality level (LQL)**.
- The LTPD is the maximum percentage of defective items that consumers are willing to tolerate.
- When the levels of lot quality specified are AQL and LTPD, the corresponding points on the OC curve are usually referred to as the producer's risk point and the consumer's risk point, respectively.

Consumer's and Producer's Risks

- Already discussed that there are two possible decisions in acceptance sampling: 1) rejecting a lot 2) accepting a lot.
- In addition, there are two situations under which the decision is made. The lot is bad or the lot is good.
- A correct decision is made if the lot is good and the sampling inspection reveals the lot to be good or if the lot is bad and the sampling inspection indicates that the lot is bad.

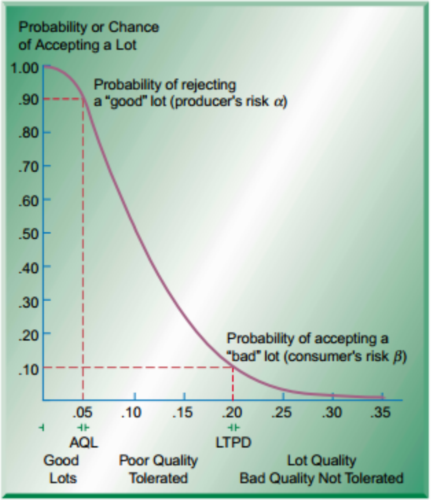
	Lot Status	
Decision	Good Lot	Bad Lot
Accept Lot	Correct	Consumer's Risk
Reject Lot	Producer's Risk	Correct

Consumer's and Producer's Risks - Contd.

- **Consumer's risk** (β) is the chance of accepting a lot containing more defects than it should (than the LTPD limit). This is the probability of making a Type II error - that is, accepting a lot that is truly "bad" (with a rejectable quality level).
- **Producer's risk** (α) is the chance of rejecting a lot containing an acceptable quality level. This is the probability of making a Type I error - that is, rejecting a lot that is "good" (with an acceptable quality level).
- Sampling plans are usually designed to meet specific levels of consumer's and producer's risk. For example, one common combination is to have a consumer's risk (β) of 10 percent and a producer's risk (α) of 5 percent, though many other combinations are possible.

Consumer's and Producer's Risks - Contd.

- The relationships among AQL, LTPD, β and α are shown below.



Average Outgoing Quality (AOQ)

- As it is observed with the OC curves, the higher the quality of the lot, the higher is the chance that it will be accepted. Conversely, the lower the quality of the lot, the greater is the chance that it will be rejected.
- Acceptance sampling programs usually require corrective action when lots are rejected. This generally takes the form of 100% inspection or screening of rejected lots, with all discovered defective items replaced with good items.
- The producer's fraction defective is called *incoming quality*. The expected fraction defective in the lot (average value of lot quality) after a particular sampling plan is called *average outgoing quality (AQL)* which is a function of the incoming quality.

Average Outgoing Quality (AOQ) - Contd.

- AOQ is the average value of lot quality that would be obtained over a long sequence of lots from a process with fraction defective p.
- Assume that all discovered defectives are replaced with good units:

$$AOQ = pP_a \left(\frac{N - n}{N} \right)$$

where p is the proportion of defective items in a lot, P_a is the probability of accepting a lot, N is the size of the lot and n is the sample size (after inspection, n items contain no defectives, because all are replaced).

- If the lot is rejected, $N - n$ items also contain no defectives and if the lot is accepted, $N - n$ items contain $p(N - n)$ defectives.

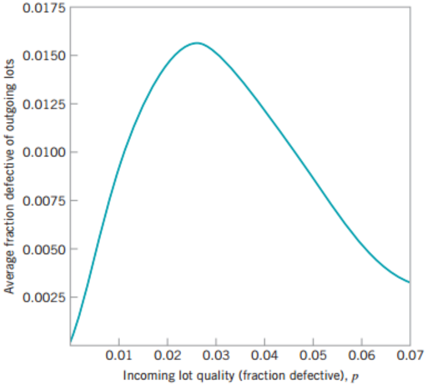
AOQ - Example

- Given $N = 10000$, $n = 89$, $c = 2$ and $p = 0.01$,
  - Find the probability of accepting a lot.
  - Find the AOQ.
- Solution:
  - $P_a = 0.9397$ (found previously)
  - $AOQ = pP_a\left(\dfrac{N-n}{N}\right) = 0.01(0.9397)\left(\dfrac{10000-89}{10000}\right) = 0.0093.$

    Thus, the average outgoing quality is 0.93% defective.

AOQ Curve

- AOQ varies as the fraction defective of the incoming lots varies.
- The curve plots AOQ against incoming lot quality. The AOQ curve for $n = 89$ and $c = 2$ is shown below.



AOQ Curve - Contd.

- When the incoming quality is very good, the average outgoing quality is also very good. In contrast, when the incoming lot quality is very bad, most of the lots are rejected and screened, which leads to a very good level of quality in the outgoing lots.

- In between these extremes, the AOQ curve rises, passes through a maximum, and descends. The maximum ordinate on the AOQ curve represents the worst possible average quality and this point is called the **average outgoing quality limit (AOQL)**.

- From the figure above $AOQL = 0.0155$. That is, no matter how bad the fraction defective, the outgoing lots will never have a worse quality level on the average than 1.55% defective.

- Note that the AOQL is an average level of quality across a large stream of lots. It does not give assurance that an isolated lot will have quality no worse than 1.55% defective.

Average Total Inspection (ATI)

- **Average total inspection (ATI)**: Another important measure relative to rectifying inspection is the total number of inspection required by the sampling program.

$$ATI = n + (1 - P_a)(N - n)$$

- If $p = 0$, the lots contain no defective items $\Rightarrow$ no lots will be rejected ($P_a = 1$) $\Rightarrow$ $ATI = n$ (the number of inspection per lot).
- If $p = 1$, the items are all defective $\Rightarrow$ every lot will be submitted to 100% inspection $\Rightarrow$ $ATI = N$.
- It is possible to draw a curve of average total inspection as a function of lot quality (p).

Example

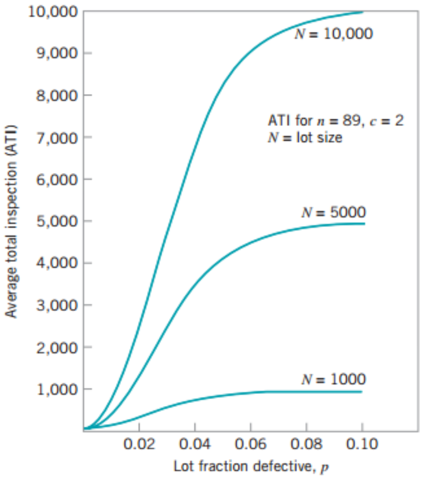
- Given $N = 10000$, $n = 89$, $c = 2$ and $p = 0.01$. Find ATI.
- Solution: $P_a = 0.9397$ (found previously)

$$ATI = n + (1 - P_a)(N - n) = 89 + (1 - 0.9397)(10000 - 89) \approx 687$$

The average number of units inspected over many lots with fraction defective $p = 0.01$ is $ATI = 687$.

ATI Curve

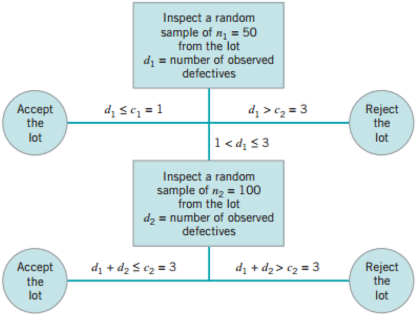
- ATI curves for n = 89, c = 2, for lot sizes of 1000, 5000, and 10000.



Double-Sampling Plan

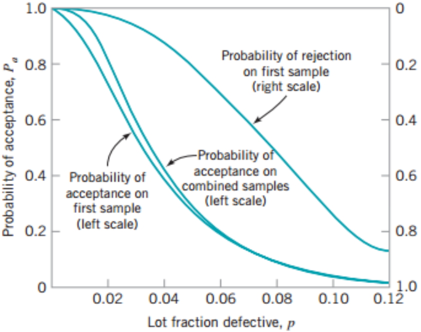
- A *double-sampling plan* is a procedure in which a second sample is required before sentencing the lot. It is defined by four parameters: $n_1 =$ sample size of the first sample, $c_1 =$ acceptance number of the first sample, $n_2 =$ sample size of the second sample and $c_2 =$ acceptance number for both samples.

- Suppose $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, and $c_2 = 3$



The OC curve for Double-Sampling Plan

- The OC curve gives P_a as a function of lot or process quality (p).



- The probability of acceptance on the combined samples (P_a) is the sum of the probability of acceptance on the first sample (P_a^I) and the probability of acceptance on the second sample (P_a^{II}).

$$P_a = P_a^I + P_a^{II}$$

Example

- For the previous example $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, and $c_2 = 3$. Find the probability of accepting a lot if $p = 0.05$.

- **Solution**: If $d_1 \leq 1$, accept the lot and if $d_1 > 3$ reject the lot. If $1 < d_1 \leq 3$, second sample is taken. Then, if $d_1 + d_2 \leq 3$, accept the lot and if $d_1 + d_2 > 3$ reject the lot.

- $P_a^I = P(d_1 \leq 1) = \sum_{d_1=0}^{1} \binom{50}{d_1} (0.05)^{d_1} (1 - 0.05)^{50-d_1} = 0.279$

- To obtain the probability of acceptance on the second sample, the number of ways the second sample can be obtained must be listed. A second sample is drawn only if there are two or three defectives on the first sample.

  - $d_1 = 2$ and $d_2 = 0$ or 1
  - $d_1 = 3$ and $d_2 = 0$

Solution - Contd.

- Thus, P_a^{II} is calculated as:

$$\begin{aligned} P_a^{II} &= P\{d_1 = 2, d_2 \leq 1\} + P\{d_1 = 3, d_2 = 0\} \\ &= P(d_1 = 2)P(d_2 \leq 1) + P(d_1 = 3)P(d_2 = 0) \\ &= \binom{50}{2}(0.05)^2(0.95)^{48}\sum_{d_2=0}^{1}\binom{100}{d_2}(0.05)^{d_2}(0.95)^{100-d_2} \\ &+ \binom{50}{3}(0.05)^3(0.95)^{57}\binom{100}{0}(0.05)^0(0.95)^{100} \\ &= 0.261(0.037) + 0.220(0.0059) \\ &= 0.0107 \end{aligned}$$

- The probability of acceptance of a lot for $p = 0.05$ is therefore $P_a = P_a^I + P_a^{II} = 0.279 + 0.0107 = 0.2897$.

Rectifying Inspection

- With double sampling, the AOQ curve is given by

$$AOQ = \frac{[P_a^I(N - n_1) + P_a^{II}(N - n_1 - n_2)]p}{N}.$$

  assuming that all defective items discovered, either in sampling or 100% inspection, are replaced with good ones.

- The average total inspection curve is given by

$$ATI = n_1 P_a^I + (n_1 + n_2)P_a^{II} + N(1 - P_a).$$

Multiple Sampling Plan

- A multiple-sampling plan is an extension of double-sampling in that more than two samples can be required to sentence a lot.
- An example of a multiple sampling plan with five stages follows.

Cumulative Sample Size	Acceptance Number	Rejection Number
20	0	3
40	1	4
60	3	5
80	5	7
100	8	9

Let d_1, d_2, d_3, d_4 and d_5 be the number of defective items in the first, second, third, fourth and fifth sample respectively.

Multiple Sampling Plan - Contd.

1. If $d_1 \leq 0$, the lot is accepted.
   If $d_1 > 3$, the lot is rejected.
   If $0 < d_1 \leq 3$, second sample is taken.

2. If $d_1 + d_2 \leq 1$, the lot is accepted.
   If $d_1 + d_2 > 4$, the lot is rejected.
   If $1 < d_1 + d_2 \leq 4$, third sample is taken.

3. If $d_1 + d_2 + d_3 \leq 3$, the lot is accepted.
   If $d_1 + d_2 + d_3 > 5$, the lot is rejected.
   If $3 < d_1 + d_2 + d_3 \leq 5$, fourth sample is taken.

4. If $d_1 + d_2 + d_3 + d_4 \leq 5$, the lot is accepted.
   If $d_1 + d_2 + d_3 + d_4 > 7$, the lot is rejected.
   If $5 < d_1 + d_2 + d_3 + d_4 \leq 7$, fiveth sample is taken.

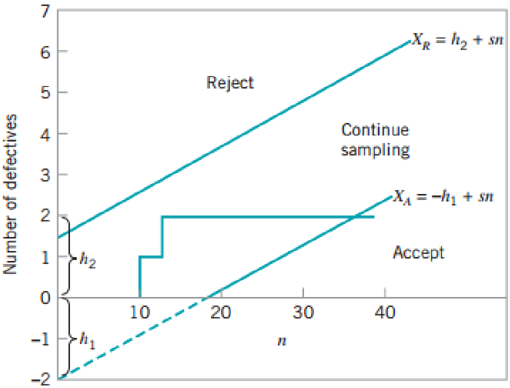
5. If $d_1 + d_2 + d_3 + d_4 + d_5 \leq 8$, the lot is accepted.
   If $d_1 + d_2 + d_3 + d_4 + d_5 \geq 9$, the lot is rejected.

Sequential Sampling Plan

- In sequential sampling, a sequence of samples from the lot are taken.
- The number of samples are allowed to be determined entirely by the results of the sampling process.
- In practice, sequential sampling can theoretically continue indefinitely, until the lot is inspected 100%.
- If the sample size inspected at each stage is one, the procedure is usually called *item-by-item sequential sampling*.
- If the sample size selected at each stage is greater than one, the process is usually called *group sequential sampling*.

Sequential Sampling Plan - Contd.

- The operation of an item-by-item sequential sampling plan is illustrated in the following figure.



Sequential Sampling Plan - Contd.

- The equations for the two limit lines for specified values of p_1, p_2, α and β are:

$$X_A = -h_1 + sn \text{ (acceptatnce line)}$$

$$X_R = h_2 + sn \text{ (rejection line)}$$

- The paramters of these lines are determined as:

$$h_1 = \frac{1}{k}\log\left(\frac{1-\alpha}{\beta}\right),\ h_2 = \frac{1}{k}\log\left(\frac{1-\beta}{\alpha}\right) \text{ and } s = \frac{1}{k}\log\left(\frac{1-p_1}{1-p_2}\right)$$

where

$$k = \log\left(\frac{p_2(1-p_1)}{p_1(1-p_2)}\right)$$

Example

- Find a sequential-sampling plan for which $p_1 = 0.01$, $p_2 = 0.06$, $\alpha = 0.05$ and $\beta = 0.10$.

$$k = \log\left(\frac{p_2(1-p_1)}{p_1(1-p_2)}\right) = \log\left(\frac{0.06(1-0.01)}{0.01(1-0.06)}\right) = 0.80066$$

$$s = \frac{1}{k}\log\left(\frac{1-p_1}{1-p_2}\right) = \frac{1}{0.80066}\log\left(\frac{1-0.05}{1-0.06}\right) = 0.028$$

$$h_1 = \frac{1}{k}\log\left(\frac{1-\alpha}{\beta}\right) = \frac{1}{0.80066}\log\left(\frac{1-0.05}{0.10}\right) = 1.22$$

$$h_2 = \frac{1}{k}\log\left(\frac{1-\beta}{\alpha}\right) = \frac{1}{0.80066}\log\left(\frac{1-0.10}{0.05}\right) = 1.57$$

$$\Rightarrow X_A = -1.22 + 0.028n \text{ (acceptatnce line)}$$
$$X_R = 1.57 + 0.028n \text{ (rejection line)}$$

Solution - Contd.

$n = 1$

$X_A = -1.22 + 0.028(1) = -1.192 \rightarrow$ The lot cannot be accepted.
$X_R = 1.57 + 0.028(1) = 1.598 \rightarrow$ The lot cannot be rejected.

$n = 2$

$X_A = -1.22 + 0.028(2) = -1.164 \rightarrow$ The lot cannot be accepted.
$X_R = 1.57 + 0.028(2) = 1.626 \rightarrow$ If both 1^{st} and 2^{nd} items are defective, the lot is rejected.

$\vdots$

$n = 45$

$X_A = -1.22 + 0.028(45) = 0.04 \rightarrow$ If all of the 45 items are nondefective, the lot is accepted.
$X_R = 1.57 + 0.028(45) = 2.83 \rightarrow$ If 3 out of 45 items are defective, the lot is rejected.

The acceptance number is the next integer $\leq X_A$, and the rejection number is the next integer $\geq X_R$. Thus, for $n = 45$, the acceptance number is 0 and the rejection number is 3.

Reliability

- Simply stated, *reliability* is quality over the long run.
- *Quality* is the condition of the product during manufacturing or immediately afterward, whereas *reliability* is the ability of the product to perform its intended function over aperiod of time.
- A product that works for a period of time is a reliable one. Since all units of a product will fail at different times, reliability is a probability.
- More precisely, reliability is the probability that a product will perform its intended function satisfactorily for a prescribed life under certain stated environmental condtions.
  - From this defintion, there are four factors associated with reliability: (1) numerical value, (2) intended function, (3) intended life and (4) environmental conditions.

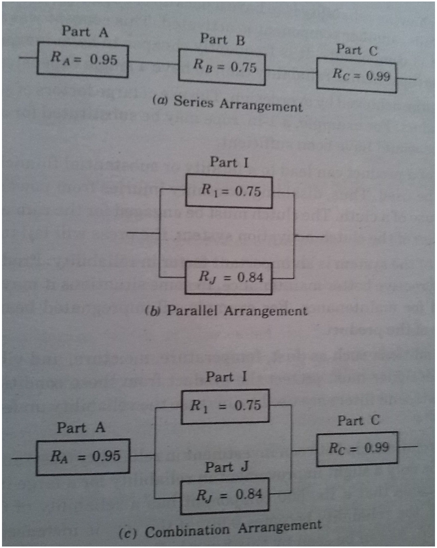
Factors associated with Reliability

1. **Numerical Value**: It is the probability that the product will not fail during a particular time. The value 0.92 represents that 92 of 100 products would function after a prescribed period of time and 8 products would fail before the prescribed period of time.
2. **Intended Function**: Products are designed for particular applications and are expected to be able to perform those applications.
3. **Intended Life**: The intended life of a products is a measure how long the product is expected to last. For example, the life of automobile tyres is specified as 40 months or 50000km. Product life is specified as a function of usage, time or both.
4. **Environmental Conditions**: A product designed to function indoors cannot be expected to function reliably outdoors in the sun, wind, moisture or precipitation, and dust.

Product and System Reliability

- Product Reliability: Increased emphasis is given to product reliability.
    - Products are more complicated. For example, at one time a washing machine was a simple device that agitated the cloths in a hot and soapy solution. Nowadays, a washing machine has different agitating speeds, temperatures, water levels and provisions to dispense a number of washing ingredients at precise times.
    - Another reason is due to automation. People are not able to manually operate the product if an automated component fails.
- System Reliability: As products become more complex (have more components), the chances of failure increases.
    - The method of arranging the components affects the reliability of the entire system.
    - Components can be arranged in a series, parallel or combination pattern.

Arrangement of Components



Arrangement of Components - Contd.

- **Series**: When components are arranged in series, the reliability of the system is the product of the individual components. For figure (a) above, the series reliability R_s, is calculated as:

  $$R_s = (R_A)(R_B)(R_C) = (0.95)(0.75)(0.99) = 0.71$$

  - As components are added to the series, the system reliability decreases. In the series arrangement, the failure of any component causes failure of the system.

- **Parallel**: For figure (b) above, the parallel reliability R_p is:

  $$R_p = 1 - (1 - R_I)(1 - R_J) = 1 - (1 - 0.75)(1 - 0.84) = 0.96$$

  - When a component fails, the product continues to function using another component until all parallel components have failed. Thus, failure of a single component is not a problem in parallel arrangement. As the number of components in parallel increases, the reliability increases.

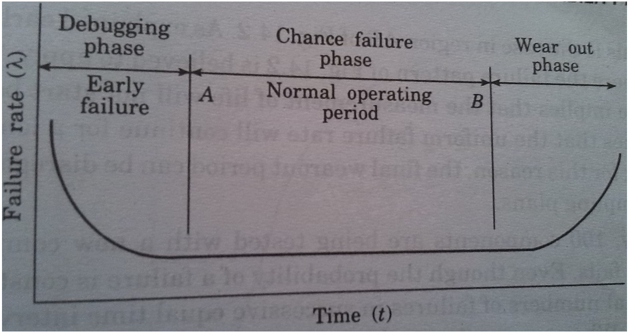
Arrangement of Components - Contd.

- **Combination**: Most complicated products are a combination of series and parallel arrangements of components.
  - This is illustrated in figure (c) above where part B is replaced by the parallel components, part I and Part J.
  - The reliability of the combination, R_c, is calculated as:

    $R_c = (R_A)(R_{I,J})(R_C) = (0.95)(0.96)(0.99) = 0.90$

Life History Curve

- Life history curve is a comparison of failure rate with time.
- It has three distinct phases: debugging phase, chance failure phase and the wear-out phase.



Life History Curve - Contd.

- The **debugging phase**: It is characterized by short life parts that cause a rapid decrease in the failure rate (Infant-Mortality Phase).
  - The Webull distribution with shaping parameter less than 1 ($\beta < 1$) is used to describe the occurrence of failures.
  - This phase is not necessary operational, hence it is rarely studied.
- The **chance failure phase** or **normal operating period**: is as a horizontal straight line, thereby making the failure rate constant. The exponential distribution is used to describe this phase of life hisory where the assumption of constant failure rate is valid.
- The **wear-out phase**: shows a sharp rise in the failure (i.e., not conforming to specification) rate.
  - Usually the normal distribution best describes this phase.
  - However, the Weibull distribution with $\beta >$ or < 3.5 can be used depending on the type of the wear-out distribution.

Reliability Tests

- Reliability testing sometimes requires destruction of the product. Testing is normally done on the end product; however, components and parts can be tested if they are causing problems.
- Life tests are of three types:
  - **Failure-Terminated**: These life test sample plans are terminated when a preassigned number of failures occur to the sample. Acceptance criteria for the lot are based on the accumulated item test times when the test is terminated.
  - **Time-Terminated**: This type of life test sampling plan is terminated when the sample obtains a predetermined test time. Acceptance criteria for the lot are based on the number of failures in the sample during the test time.
  - **Sequential**: In this life testing plan, neither the number of failures nor the time required to reach a decision are fixed in advance. Instead, decisions depend on the accumulated results of the life test.

Reliability Tests - Contd.

- Tests are based one one or more of the following characteristics:
    - *Mean life* - the average life of the product.
    - *Failure rate* - the percentage of failures per unit time.
    - *Hazard rate* - the instantaneous failures rate at a specified time.
    - *Reliable life* - the life beyond which some specified portion of the items in the lot will survive.

WISH U A HAPPY AND JOYFULL LIFE IN YOUR FUTURE CAREER!!!